Dinamika porazdelitve energije in adiabatske invariante v homogenih časovno odvisnih Hamiltonskih sistemih

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Če se parametri sistema, katerega dinamika je ergodična na energijski lupini, spreminjajo počasi glede na ergodično časovno skalo, potem se sistem giblje tako, da je volumen znotraj energijske lupine, $\Omega(t)$, približna konstanta gibanja oziroma adiabatska invarianta. Znana posledica tega je enačba stanja $V\,T^{f/2} =$ konstanta za adiabastke procese v idealnih plinih.

Kaj pa v primeru, da dinamika sistema ni egodična na energijski lupini? V tem primeru $\Omega(t)$ ni konstanta gibanja, ampak ali obstaja neka druga količina, ki je? Odgovor je pritrdilen vsaj za homogene Hamiltonske sisteme (na primer biljarde).

Volumen faznega prostora znotraj energijske lupine v Hamiltonskih sistemih je oblike $\Omega(t) = E^{\gamma}\,\Sigma(t)$, kjer je E energija, γ nek eksponent in $\Sigma(t)$ nek časovno odvisni geometrijski faktor. Teoretično bom pokazal, da je količina $\Sigma(t)/\left\langle E^{-\gamma}\right\rangle$, kjer $\langle\rangle$ predstavlja povprečenje, adiabatska invarianta, neodvisna od dinamičnih lastnosti sistema. Še več, pokazal bom, da v primeru, ko je variacija sistema periodična, momenti porazdelitve energije, $\langle E^n\rangle$, kjer $n \in \mathbb{R}$, naraščajo exponentno za vse n, ki zadoščajo $(n < -\gamma)$ in $(n > 0)$, in exponentno padajo za $(-\gamma < n < 0)$. Za $n = 1$ sledi eksponentno Fermijevo pospeševanje [1].

Reference

[1] B. Batistić, *Phys. Rev. E* **89** (2014) 022912.

Evolution of energy distribution and adiabatic invariants in homogeneous time-dependent Hamiltonian systems

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If a parameter of a system whose dynamics is ergodic on an energy shell is varied slowly compared to an ergodic time scale, then the system evolves in such a way that a volume of the phase space enclosed by the corresponding energy shell, $\Omega(t)$, is an approximate constant of motion or adiabatic invariant. A well known consequence of this is the equation of state $V T^{f/2} =$ constant for an adiabatic process in the ideal gas.

What if dynamics of a system on an energy shell is not ergodic? In this case $\Omega(t)$ is not an adiabatic invariant, but does there exist some other quantity that is? The answer is yes at least for homogeneous Hamiltonian systems (for example billiards).

In homogeneous Hamiltonian systems a volume of the phase space enclosed by an energy shell takes the form $\Omega(t) = E^{\gamma}\,\Sigma(t)$, where E is the energy, γ is some exponent and $\Sigma(t)$ is some time-dependent geometrical factor. I shall show theoretically that the quantity $\Sigma(t)/\langle E^{-\gamma}\rangle$, where $\langle\rangle$ denotes the averaging over an energy distribution, is an adiabatic invariant, independent of the dynamical properties of the system. Additionally, I shall show that if the parameters of the system vary periodically, then, in general, the moments of the energy distribution, $\langle E^n\rangle$, where $n \in \mathbb{R}$, grow exponentially with the number of oscillations for all exponents n that satisfy $(n < -\gamma)$ and $(n > 0)$, and decrease exponentially for $(-\gamma < n < 0)$. In particular, for $n = 1$ this implies exponential Fermi acceleration [1].

References

[1] B. Batistić, *Phys. Rev. E* **89** (2014) 022912.

Entropic Distance Evolution for Growing Networks

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In a balanced version of decay and growth processes the simplest linear master equation with constant and/or linear rates in the connection degree arrives at the Poissonian, Bernoulli, negative binomial and the Pólya distribution. The large n limit is a continuous diffusion model with x-dependent rates. Here well-established proofs of entropy growth are known.

On the other hand, in one-sided, avalanche-type only-growth processes the same simplest assumptions about the elementary rate lead to the geometrical (exponential) distribution and to a Waring distribution having a power-law tail for large n, respectively. The continuous version of the master equation looks flow-like. The approach to such stationary distributions in terms of an entropic distance is given for a certain class of rates.

References

[1] T. S. Biró, Z. Néda, *arxiv* **1606.05737** (2016).

[2] T. S. Biró, Z. Schram; *Entropy* **18** (2016) 2.

Detekcija narave bozonskih ekscitacij s pomočjo visoko - energijskih nosilcev naboja

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Obravnaval bom problem detekcije narave bozonskih ekscitacij, ki so močno sklopljene s foto - vzbujenimi nosilci naboja. Pokazal bom, da lahko s pomočjo že uveljavljenih časovno ločljivih spektroskopskih metod ločimo sklopitev z običajnimi bozonskimi prostostnimi stopnjami, kot so npr. fononi ter bozoni s t.i. trdo sredico, kot so magnoni. Metoda sloni na dejstvu, da imajo fononi neomejen ekscitacijski spekter. Zato lahko absorbirajo v principu neomejeno količino energije. Posledično je relaksacijska dinamika skoraj neodvisna od energije foto asborbiranega fotona oziroma od intenzitete laserskega žarka. Bozoni s trdo sredico pa imajo omejen ekscitacijski spekter zato je relaksacijska dinamika foto vzbujenega naboja, ki poteka preko bozonov s trdo sredico, močno odvisna od energije absorbiranega fotona oziroma intenzitete laserskega žarka.

References

[1] J. Kogoj, M. Mierzejewski, and J. Bonča, arXiv:1606.08669 (2016). Accepted for publication in *Physical Review Letters.*

Nature of bosonic excitations revealed by high - energy charge carriers

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We address a long standing problem concerning the origin of bosonic excitations that strongly interact with charge carriers. We show that the time - resolved pump - probe experiments are capable to distinguish between regular bosonic degrees of freedom, e.g. phonons, and the hard-core bosons, e.g., magnons. The ability of phonon degrees of freedom to absorb essentially unlimited amount of energy renders relaxation dynamics nearly independent on the absorbed energy or the fluence. In contrast, the hard core effects pose limits on the density of energy stored in the bosonic subsystems resulting in a substantial dependence of the relaxation time on the fluence and/or excitation energy. Very similar effects can be observed also in a different setup when the system is driven by multiple pulses.

References

[1] J. Kogoj, M. Mierzejewski, and J. Bonča, arXiv:1606.08669 (2016). Accepted for publication in *Physical Review Letters.*

Aranžmaji defektov v holesteričnih kapljicah

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Kapljice tekočega kristala v suspenziji lahko zavzamejo mnogo različnih struktur, ki so odvisne od lastnosti izbranega materiala in površinskega sidranja. Posebno s kiralnim nematikom (holesterikom) lahko ustvarimo kapljice z zapletenimi defektnimi strukturami [1] kot tudi urejene čebulaste strukture [2,3].

Predstavil bom zgradbo kapljic kiralnega nematika z zmerno kiralnostjo. Pravokotno površinsko sidranje povzroči frustracijo nematskega direktorja, zato namesto enakomerne plastovite vijačne strukture dobimo mehurčke lokalizirane kiralnosti. Pri hitrem ohlajanju dobimo veliko struktur z večjim številom točkastih defektov, med katerimi smo našli tudi defekte višjega topološkega naboja -2 in -3, ki do sedaj še niso bili opaženi. Ti defekti se sestavijo v različne aranžmaje, za katere veljajo posebna pravila [4]. Za opazovanje notranje zgradbe kapljic smo izpopolnili metodo fluorescentne konfokalne polarizacijske mikroskopije, s katero smo pridobili tridimenzionalno direktorsko sliko, brez katere zapletenih struktur ne bi razložili [5].

Reference

[1] D. Seč, S. Čopar & S. Žumer, Nat. Commun. **5** (2014) 3057.

[2] J. Bajc, P. P. Crooker & S. Žumer, Liquid Crystals Today **7** (1997) 1.

[3] M. Humar & I. Muševič, Opt. Express **18** (2010) 26995.

[4] G. Posnjak, S. Čopar & I.Muševič, *Hidden topological constellations and polyvalent charges in chiral nematic droplets*, Nat. Commun. (sprejeto).

[5] G. Posnjak, S. Čopar & I.Muševič, Sci. Rep. **6** (2016) 26361.

Defect constellations in cholesteric droplets

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Liquid crystalline droplets in a suspension can form various structures, which depend on the properties of the chosen liquid crystal and surface achnoring. A chiral nematic (cholesteric) can contain complex defect structures [1] or ordered onion-like structures [2,3].

I will present structure of chiral nematic droplets with moderate chirality. Homeotropic surface anchoring frustrates the nematic director, which instead of a uniform layer-like helical structure forms bubbles of localized chirality. With quenching, we obtain various structures with a larger number of point defects, which include defects of higher topological charges -2 and -3, which have never been observed before. These defects form different constellations, which follow certain rules [4]. For observation of the internal structure of these droplets, we improved the method of flourescent confocal polarizing microscopy, which we used to obtain three-dimensional director field image, essential to discern the complex structures [5].

References

[1] D. Seč, S. Čopar & S. Žumer, Nat. Commun. **5** (2014) 3057.
[2] J. Bajc, P. P. Crooker & S. Žumer, Liquid Crystals Today **7** (1997) 1.
[3] M. Humar & I. Muševič, Opt. Express **18** (2010) 26995.
[4] G. Posnjak, S. Čopar & I. Muševič, *Hidden topological constellations and polyvalent charges in chiral nematic droplets*, Nat. Commun. (accepted).
[5] G. Posnjak, S. Čopar & I. Muševič, Sci. Rep. **6** (2016) 26361.

Comets in extrasolar system

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The aim of our study is to investigate the dynamics of possible comets in the extrasolar planetary system HD 10180. This investigation is motivated by the discovery of previous families of exocomets in various systems, especially β Pictoris, as well as in at least ten other systems. Detailed theoretical studies about the formation and evolution of star–planet systems indicate that exocomets should be quite common, i.e., including systems with stars of different spectral types. Further observational results are expected in the foreseeable future, in part due to the availability of the *Large Synoptic Survey Telescope*. Nonetheless, the Solar System represents the best studied example for comets, thus serving as a prime motivation for investigating comets in the HD 10180 system as well. HD 10180 is strikingly similar to the Sun. In this context we also tried to distinguish cometary families like in our Solar System but it turned out that there is no clear distinction between families. This system contains six confirmed planets and (at least) two additional planets subject to final verification. In our studies, we consider comets of different inclinations and eccentricities (including comets in retrograde orbits) and find an array of different outcomes such as encounters with planets, captures, escapes, and secular comet–planet resonances. Comets with relatively large eccentricities are able to enter the inner region of the system facing early close planetary encounters. Stable comets experience long-term evolution of orbital elements, as expected. Generally, theoretical and observational studies of exoplanets have a large range of ramifications, involving the origin, structure and evolution of systems as well as the proliferation of water and prebiotic compounds to terrestrial planets, which will increase their chances of being habitable.

References

[1] R. Brasser, R. *Astronomy and Astrophysics* **492** (2008) 251.

[2] B. Loibnegger R. Dvorak and M. Cuntz *Astronomical Journal* submitted

[3] N. V. Cook D. Ragozzine M. Granvik,and D.C.Stephens, *Astrophysical Journal* **825** (2016) 51.

Anomalije v razpadih mezonov B

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V poskusih v tovarnah mezonov B in na LHCb so opazili zanimive učinke kršitve univerzalnosti leptonskiih okusov. Izmerjena razmerja $BR(B \to D^{(*)}\tau\nu_\tau)/BR(B \to D(*)\mu\nu_\mu)$ in $R_K = BR(B \to K\mu^+\mu^-)/BR(B \to Ke^+e^-)$ v področju majhnih dileptonskih mas, se ne ujemata z napovedi Standardnega modela na nivoju od 3 standardne devijacije Razširiteve Standardnega modela katere vsebuje dodatni umeritveni bozon ali leptokvark lahko pojasnijo obstoječe uganke. Predstavila bom kako skalarni in vektorski leptokvarki lahko razložijo anomalije mezonov B, brez bistvene spremembe drugih opaljivki.

Reference

[1] I. Doršner, S. Fajfer, A. Greljo, J. F. Kamenik and N. Košnik, *Phys. Rept.* **641** (2016) 1.

[2] D. Bečirević, S. Fajfer and N. Košnik, *Phys. Rev. D* **92** (2015) 1, 014016.

[3] S. Fajfer, I. Doršner, N. Košnik and I. Nišandžić, *JHEP* **1311** (2013) 084.

[4] S. Fajfer and N. Košnik, *Phys. Lett.* B **755** (2016) 270I.

[5] S. Fajfer and N. Košnik, *Eur. Phys. J.C* **75** (2015) 567.

Anomalies in B meson decays

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B-physics experiments at B-factories and at LHCb pointed out at very intriguing effects of lepton flavour universality violation. The measured ratio of $BR(B \to D^{(*)} \to \tau\nu_\tau)/BR(B \to D^{(*)}\mu\nu_\mu)$ and $R_K = BR(B \to K\mu^+\mu^-)/BR(B \to Ke^+e^-)$ at low dilepton invariant mass region disagree at the level of 3 σ from the Standard Model predictions. The extension of the Standard Model containing additional gauge boson or leptoquarks might explain the observed anomalies. I discuss how scalar or vector leptoqaurks might resolve all B meson puzzles, without significant modification of other observables.

References

[1] I. Doršner, S. Fajfer, A. Greljo, J. F. Kamenik and N. Košnik, *Phys. Rept.* **641** (2016) 1.

[2] D. Bečirević, S. Fajfer and N. Košnik, *Phys. Rev. D* **92** (2015) 1, 014016.

[3] S. Fajfer, I. Doršner, N. Košnik and I. Nišandžić, *JHEP* **1311** (2013) 084.

[4] S. Fajfer and N. Košnik, *Phys. Lett.* B **755** (2016) 270I.

[5] S. Fajfer and N. Košnik, *Eur. Phys. J.C* **75** (2015) 567.

Integrabilnost v polinomskih sistemih NDE

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Problem integrabilnosti sestoji iz določitve lokalnih ali globalnih prvih integralov in je eden od glavnih odprtih problemov v kvalitativni teoriji diferencialnih sistemov. Bistven del teorije integrabilnosti NDE je posvečen študiju lokalne integrabilnosti dvodimenzionalnih analitičnih sistemov diferencialnih enačb (dvodimenzionalnih analitičnih vektorskih polj) v okolici singularne točke tipa center ali fokus. Predstavila bom pristop k študiju problema integrabilnosti v dvodimenzionalnih polinomskih sistemih. Nato bomo poiskali integrabilne sisteme znotraj družine sistemov s homogenimi nelinearnostmi stopnje pet.

Reference

[1] B. Ferčec, J. Giné, V. Romanovski, V. Edneral, *Journal of mathematical analysis and applications* **434** (2016) L894-L914.

Integrability in polynomial systems of ODE's

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The integrability problem consists in the determination of local or global first integrals and is one of the main open problems in the qualitative theory of differential systems. An essential part of the theory of integrability of ODE's is devoted to studying local integrability of two dimensional analytic systems of differential equations (two dimensional analytic vector fields) in a neighborhood of a singular point of center or focus type. In this talk we describe an approach for studying integrability in two dimensional polynomial systems. Then, we find integrable systems within a family of systems with quintic homogeneous nonlinearities.

References

[1] B. Ferčec, J. Giné, V. Romanovski, V. Edneral, *Journal of mathematical analysis and applications* **434** (2016) L894-L914.

S pristopom večplastnih mrež do razumevanja organizacijskih principov v populaciji pankreasnih celic beta

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Celice beta v Langerhansovih otočkih v trebušni slinavki izločajo inzulin in tvorijo kompleksen sincicij z ne-trivialno povezanimi elementi, ki so intrinzično nelinearni in zelo heterogeni. Posamezne celice beta izkazujejo tudi večmodalno ritmično aktivnost, ki je posledica povratnih interakcij različnih oscilatornih podsistemov, in sicer glikolitskega, mitohondrijskega in električno-kalcijevega [1]. Dodatno stopnjo dinamične kompleksnosti vnesejo medcelične povezave, ki zagotavljajo normalno delovanje. Kvantifikacija kolektivne aktivnosti tega večceličnega sistema je zatorej težka naloga in nedavno so se v tem kontekstu za uspešne izkazale statistične metode s področja teorije kompleksnih mrež [2-4]. Vendar zaradi nestacionarnega obnašanja populacije celic beta, na katerega vplivajo različne vrste medceličnih interakcij in se s časom tudi spreminja, predstavlja standardni mrežni pristop morda preveliko poenostavitev. V zadnjih letih se za opis in analizo takšnih večdimenzionalnih kompleksnih sistemov vse pogosteje uporablja formalizem, ki temelji na teoriji večplastnih mrež, ter postaja vse bolj prepoznavna smer znanstveno-raziskovalnega razvoja [5]. V predavanju bom najprej podal pregled osnovnih principov teorije večplastnih mrež. V nadaljevanju bom pokazal nekaj različnih primerov uporabe večplastnih mrež, na podlagi katerih se lahko uspešno prikaže in preučuje kompleksna signalizacija in komunikacija v Langerhansovih otočkih.

Reference

[1] J. Ren, A. Sherman, R. Bertram, P. B. Goforth, C. S. Nunemaker, C. D. Waters, L. S. Satin, *Am. J. Physiol.* **305** (2013) E805.

[2] A. Stožer, M. Gosak, J. Dolenšek, M. Perc, M. Marhl, M. S. Rupnik, D. Korošak, *PLoS Comput. Biol.* **9** (2013) e1002923.

[3] R. Markovič, A. Stožer, M. Gosak, J. Dolenšek, M. Marhl, M. S. Rupnik, *Scientific Reports* **5** (2015) 7845.

[4] M. Gosak, A. Stožer, R. Markovič, J. Dolenšek, M. Marhl, M. S. Rupnik, M. Perc, *Chaos* **25** (2015) 073115.

[5] S. Boccaletti, *et al.*, *Physics Reports* **544** (2014) 1.

Multilayer network approaches to understanding the organizing principles of pancreatic beta cell populations

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Insulin-secreting beta cells within the pancreatic islets of Langerhans form a complex syncytium with non-trivially interconnected elements that are intrinsically nonlinear and heterogeneous. Individual beta cells exhibit a multimodal rhythmic activity, which is a result of feedback interactions of different oscillatory subsystems, such as the glycolytic, mitochondrial and electrical/calcium components [1]. An even higher level of dynamical complexity is introduced by intercellular coupling, a necessary component for normal function. Quantifying the collective activity of this complex multicellular system is a challenge and recently statistical methods originating from the theory of complex networks have proven to be a suitable framework in this respect [2-4]. However, due to the nonstationary behavior of beta cell populations, that changes with time and is governed by multiple types of intercellular interactions, the standard network approach might be an oversimplification. In the last years, the multilayer network formalism has been proposed as a general theoretical framework for the description and analysis of such multi-dimensional complex systems and is acquiring more and more prominence in terms of a new research direction [5]. In the seminar, I shall first review the basic principles of the multilayer network theory. Then, I will show different examples of how the multilayer network approach can be used to visualize and gain new insights into the complex signalization and communication patterns in islets of Langerhans.

References

[1] J. Ren, A. Sherman, R. Bertram, P. B. Goforth, C. S. Nunemaker, C. D. Waters, L. S. Satin, *Am. J. Physiol.* **305** (2013) E805.

[2] A. Stožer, M. Gosak, J. Dolenšek, M. Perc, M. Marhl, M. S. Rupnik, D. Korošak, *PLoS Comput. Biol.* **9** (2013) e1002923.

[3] R. Markovič, A. Stožer, M. Gosak, J. Dolenšek, M. Marhl, M. S. Rupnik, *Scientific Reports* **5** (2015) 7845.

[4] M. Gosak, A. Stožer, R. Markovič, J. Dolenšek, M. Marhl, M. S. Rupnik, M. Perc, *Chaos* **25** (2015) 073115.

[5] S. Boccaletti, *et al.*, *Physics Reports* **544** (2014) 1.

Generalised global symmetries and dissipative magnetohydrodynamics

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The conserved magnetic flux of $U(1)$ electrodynamics coupled to matter is associated with a generalised global symmetry. I will discuss the realisation of such a symmetry at finite temperature and develop the hydrodynamic theory describing fluctuations of a conserved 2-form current around thermal equilibrium. This can be thought of as a systematic derivation of relativistic magnetohydrodynamics, constrained only by symmetries and effective field theory. By constructing the entropy current, I will then show that at first order in derivatives, there are six dissipative transport coefficients. This will enable me to present a universal definition of resistivity in a theory of dynamical electromagnetism and derive a direct Kubo formula for the resistivity in terms of correlation functions of the electric field operator. I will also discuss fluctuations and collective modes, presenting novel expressions for the dissipative widths of magnetosonic and Alfvén modes. Finally, I will demonstrate that a non-trivial truncation of the theory can be performed at low temperatures compared to the magnetic field: this theory has an emergent Lorentz invariance along magnetic field lines, and hydrodynamic fluctuations are now parametrised by a fluid tensor rather than a fluid velocity. Throughout, no assumption will be made of weak electromagnetic coupling; thus, this theory should be able to describe new physics of dense electromagnetic plasmas.

References

[1] S. Grozdanov, D. M. Hofman and N. Iqbal, "Generalized global symmetries and dissipative magnetohydrodynamics," arXiv:1610.07392 [hep-th].

Časovna evolucija periodično brcanih kvantnih sistemov v Heisenbergovi sliki

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Zanimiva družina kvantnih mnogodelčnih sistemov izven ravnovesja so Floquetovi sistemi, t. j. sistemi kvantih delcev v časovno perodičnem potencialu. S spreminjanjem parametrov periodičnega vzbujanja lahko močno vplivamo na lastnosti sistema in opazimo zanimivo obnašanje, kot so recimo prehodi med integrabilnimi in kaotičnimi režimi, lokalizacija in prehodi v eksotične faze snovi.

V nekaterih modelih s preprosto časovno odvisnostjo lahko numerično učinkovito ocenimo dinamične lastnosti sistema. Na posebnem modelu bom predstavila rezultate, ki kažejo na eksponentno dolge časovne skale, na katerih opazimo padanje korelacij. To je posledica obstoja količine, ki sicer ni ohranjena, temveč se zelo dobro ujema z efektivno Hamiltonovo funkcijo sistema.

Heisenberg picture time-evolution of periodically kicked quantum systems

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Floquet systems, i. e. systems with time periodic driving, are an interesting class of out-of-equilibrium quantum many body systems. By modifying the driving parameters, the systems can exhibit many interesting behaviours such as transitions between integrability and non-integrability, dynamical localization and transitions into exotic phases of matter.

For some models with particularly simple time dependence, we are able to effectively calculate a numerical bound for some dynamical properties. I am going to show some results that imply exponentially long time scales for correlation decay. This is a consequence of the existence of a quantity that is not conserved, but approximates the effective Hamiltonian of the system.

Belle II - eksperiment v nastajanju

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V predavanju bom najprej predstavil motivacijo za naslednjo generacijo poskusov v fiziki težkih kvarkov in leptonov, iskanju signalov fizikalnih procesov, ki jih ne moremo zadovoljivo popisati v okviru Standardnega modela, teorije osnovnih delcev in njihovih interakcij. Obdelali bomo pogoje, ki jim mora zadoščati detektor za načrtovane študije. Sledil bo pregled najbolj zanimivih trenutkov pri nastajanju novega detektorja, predstavil pa bom tudi načrte za bodočnost.

Belle II - an experiment in the making

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In the talk we will first discuss the motivation for the next generation of experiments in heavy flavour physics, searches for phenomena that cannot be accommodated in the Standard model, the theory of elementary particles and their interactions. We will then discuss the requirements for the next generation detector, and present some interesting steps in the construction of the Belle II spectrometer. We will finish by our plans for the future.

(Lattice) QCD - what else!

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Given QCD as the fundamental Quantum Field Theory of strong interactions we have to find non-perturbstive methods to obtain the low energy properties of hadrons, their masses, scattering behavior and structure. The formulation on a Euclidean space-time lattice does this job. I will concentrate on the tools to determine the predicted (actually: postdicted) hadron mass spectrum and scattering properties in the resonance region, where considerable progress has been made.

References

[1] C. B. Lang, D. Mohler, S. Prelovsek, and M. Vidmar. *Phys. Rev. D* **84** (2011) 054503; Erratum Phys. Rev. D **89** (2014) 059903(E); arXiv:1105.5636

[2] C. B. Lang and V. Verduci. *Phys. Rev. D* **87** (2013) 054502; arXiv:1212.5055 [hep-lat].

[3] C. B. Lang, L. Leskovec, M. Padmanath and S. Prelovsek. arXiv:1610.01422

[4] C. B. Lang, L. Leskovec, D. Mohler and S. Prelovsek *JHEP* **09** (2015) 089; arXiv:1503.035363 [hep-lat]

[5] C. B. Lang, D. Mohler, S. Prelovsek, and R. M. Woloshyn. *Physics Letters B* **750** (2015)17; arXiv:1501.01646 [hep-lat]

[6] M. Padmanath, C. B. Lang, and S. Prelovsek *PoS* LATTICE 2015 (2015) 092; arXiv:1510.09150 [hep-lat]

[7] M. F. M. Lutz, J. Sören Lange, M. Pennington, et al.. EMMI Rapid Reaction Task Force meeting "Resonances in QCD", GSI October 12-14, 2015; *Nucl. Phys.* **A948** (2016) 93-105; arXiv:1511.09353 [hep-ph]

[8] C. B. Lang. Invited talk at the XVI International Conference on Hadron Spectroscopy, September 13-18, 2015, Newport News; *AIP Conference Proceedings* **1735**, 020002 (2016); doi: 10.1063/1.4949370, arXiv:1512.05545 [nucl-th]

[9] C. B. Lang, D. Mohler and S. Prelovsek. *Phys.Rev. D* (2016) in print; arXiv:1607.03185 [hep-lat]

Neravnovesni spinski transport v anizotropni Heisenbergovi spinski verigi

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Znano je, da je model anizotropne Heisenbergove spinske verige integrabilen. Za režim $\Delta < 1$ je s pomočjo Mazurjeve neenakosti dokazano, da je spinski transport balističen, lastnosti spinskega transporta v režimu $\Delta \geq 1$ pa ostajajo odprto vprašanje. V zadnjih letih smo bili na to temo priča večjemu številu numerišnih simulacij, ki pa so predvsem omejene s kratkimi časi oziroma majhnimi sistemi. Predstavil bom nove numerične rezultate, kjer s pomočjo metode tDMRG spremljamo časovni razvoj izbranih nehomogenih začetnih stanj sistema do relativno dolgih časov. Na podlagi časovne odvisnosti lokalnih spinskih tokov in skaliranja spinskih profilov pri različnih časih lahko iz dobljenih rezultatov določimo, da je v režimu $\Delta = 1$ transport superdifuzijski, medtem ko pri $\Delta > 1$ preidemo v difuzijski režim.

Out-of-equilibirum spin transport in the anisotropic Heisenberg spin chain

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We know that the anisotropic Heisenberg spin chain model is integrable. It was proven, through the use of the Mazur inequality, that the spin transport in the $\Delta < 1$ regime is ballistic, while the properties of spin transport in the anisotropic Heisenberg spin chain in the $\Delta \geq 1$ regime remain an open question. This problem has seen many numerical simulations in recent years, but they are mostly limited to short times or small systems. I will present new numerical results where we use the tDMRG method to study the time evolution of a set of inhomogeneous inital states of the system up to relatively long times. Based on these results we show that the spin transport at $\Delta = 1$ is superdiffusive, while the $\Delta > 1$ regime gives rise to diffusive transport. This is done by examining the time dependence of local spin currents as well as the scaling of spin profiles at various times.

Uporaba pristopov s področja kompleksnih mrež za rekonstrukcijo in analizo anatomije ožilja

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Natančne in celovite meritve anatomije mikrovaskularnega omrežja so izjemno pomembne za proučevanje zdravega in patološkega razvoja ožilja. Ključni mehanizem, ki kroji angiogenezo ožilja, je Wnt/Frizzled signalna pot skupaj s pojavom planarne celične polarnosti (Wnt/PCP) [1]. Z namenom podrobnejše analize vpliva te signalne poti na žilno strukturo, in to na makroskopski ravni celega organa, uporabimo metodo slikanja ožilja ledvice z mikroračunalniško tomografijo s sledečo računalniško 3D rekonstrukcijo posnetega žilnega omrežja. Za slednje uporabimo metodološko nove pristope, ki temeljijo na orodjih s področja teorije kompleksnih mrež. Predlagana metoda zagotavlja visoko stopnjo zanesljivosti in vključuje tudi algoritme za dekompozicijo 3D omrežij v podatkovna poddrevesa, multifraktalno analizo in analizo vzorcev razvejanosti ožilja ter karakterizacijo nekaterih globalnih in lokalnih strukturnih značilnosti (npr. dolžina, površina, koti razcepov in hierarhična organizacija). Primerjava rezultatov ožilja, pridobljenih iz zdravih miši s tistimi iz mutiranih miši z okvarjeno Wnt/PCP signalno potjo, razkriva očitne razlike v mnogih parametrih ožilja. Naše ugotovitve potrjujejo, da geni, ki so vključeni v Wnt/PCP signalno pot, na zelo specifičen način sooblikujejo strukturo ožilja in imajo tako

posebno vlogo pri arterijski morfogenezi in posledično vplivajo tudi na funkcionalno učinkovitost ožilja [2]. Boljše razumevanje Wnt/PCP signalne poti in z njo povezane angiogeneze je tako ključnega pomena pri iskanju načinov nadzora bodisi v smislu zaviranja ali pospeševanja procesa angiogeneze. Slednje pa bi lahko prispevalo k oblikovanju novih metod zdravljenja številnih najbolj ogrožajočih bolezni moderne dobe, od odpovedi srca do sladkorne bolezni in raka.

Reference

[1] M. Zerlin, *et al.*, *Angiogenesis* **11** (2008) 63-9.

[2] R. Markovič, *et al.*, poslano v *Plos ONE*.

Utilizing complex network-based approaches for the reconstruction and analysis of the vascular network anatomy

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Precise and comprehensive measurements of microvascular network anatomy are one of the crucial steps for the analysis of normal as well as pathologic vascular development, and is of paramount importance for the understanding of several aspects of microcirculation. The key mechanism that governs the angiogenesis is the Wnt/Frizzled planar cell polarity pathway (Wnt/PCP) [1]. To examine its role in more detail, especially on the level of whole vasculatures, we combine microcomputed tomography imaging of kidney vasculatures and the subsequent 3D reconstruction of vascular networks. For the later we use novel methodological approaches based on the realms of the complex network theory. The proposed method ensures a high degree of accuracy of the reconstructed vascular networks. Further computational approaches include the decomposition of 3D networks into subtree data structures, multifractal and branching pattern analyses, and the characterization of several global and local structural features (i.e. length, surface, bifurcation angles and hi-

erarchical organization). Comparing the results obtained from normal (wild-type) mice with those from mutant mice with impaired Wnt/PCP pathways reveals obvious differences in several parameters of the vascular network. Our findings thereby confirm that the Wnt/PCP related genes deeply exert specific patterning role in arterial vessel morphogenesis that may determine its functional efficiency [2]. A better understanding of the Wnt/PCP and the related angiogenesis is crucial for finding ways of controlling, either inhibiting or promoting the process of angiogenesis, which might contribute to healing some of the most threatening diseases of modern era ranging from hearth failures to diabetes and cancer.

References

[1] M. Zerlin, *et al.*, *Angiogenesis* **11** (2008) 63-9.

[2] R. Markovič, *et al.*, submitted to *Plos ONE*.

Transport in lokalnost

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Narava transporta je eno od osrednjih vprašanj pri obravnavi mnogodelčnih kvantnih sistemov. Že na primeru Heisenbergove XXZ verige vidimo pester nabor režimov – od balističnega, anomalnega do difuzijskega. V predstavitvi bom povezal transportne koeficiente z lokalnimi ohranitvenimi zakoni in predstavil konkretne rezultate za primer Heisenbergove verige.

Transport and locality

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The nature of transport is one of the central questions in the study of many-body quantum systems. The systems can exhibit wide range of transport phenomena, such as ideal, anomalous and diffusive transport, as exemplified by the paradigmatic Heisenberg XXZ model. In the presentation I will make a connection between the transport coefficients and local conservation laws and show some exact results for the Heisenberg XXZ model.

Stremenje k pravičnosti v človeških ultimatih

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Predstavljajmo si dva igralca, ki si morata razdeliti vsoto denarja. Eden predlaga delitev, drugi pa se lahko s slednjo strinja ali pa ne. Če obstaja konsenz se denar razdeli kot predlagano v ultimatu, v nasprotnem primeru pa oba igralca ostaneta praznih rok. Eksperimenti z igro ultimatov so pokazali, da smo ljudje zelo občutljivi na pravičnost. Nepravične ponudbe so vselej redke, in verjetnost, da z njimi uspemo je minimalna. Teoretične študije igre ultimatov so tradicionalno privzele zvezen prostor strategij, in izkazalo se je, da empatija in prostorsko omejene interakcije podpirajo pravičnost v tem okvirju. Toda, evolucijske igre z zveznimi strategijami pogosto skrijejo realno kompleksnost problema, saj so rešitve, ki bi bile plod prostorskih vzorcev na interakcijski mreži, nestabilne. Diskretne strategije, po drugi strani, pa v igri ultimatov odprejo vrata do fascinantno kompleksne časovno-prostorske dinamike, ki razkrije dejanske težave pri stremenju k pravičnosti v človeških ultimatih [1,2].

Reference

[1] A. Szolnoki, M. Perc, G. Szabo, Accuracy in strategy imitations promotes the evolution of fairness in the spatial ultimatum game, *EPL* **100** (2012) 28005.

[2] A. Szolnoki, M. Perc, G. Szabo, Defense mechanisms of empathetic players in the spatial ultimatum game, *Physical Review Letters* **109** (2012) 078701.

Striving towards fairness in human ultimatums

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Imagine two players having to share a sum of money. One proposes a split, and the other can either agree with it or not. If there is an agreement, the sum is shared according to the ultimatum. If not, both players remain empty handed. Seminal experiments on the ultimatum game have revealed that humans are remarkably fond of fair play. When asked to share something, unfair offers are rare and their acceptance rate is small. Theoretically, the ultimatum game has been studied with continuous strategies, and it has been shown that empathy and spatiality may lead to the evolution of fairness. However, evolutionary games with continuous strategies hide the true complexity of the problem, because solutions that would be driven by pattern formation are unstable. Discrete strategies in the ultimatum game open the gate to fascinatingly rich dynamical behavior [1,2]. The obtained phase diagram reveals the hidden complexity behind the pursuit of human fair play.

References

[1] A. Szolnoki, M. Perc, G. Szabo, Accuracy in strategy imitations promotes the evolution of fairness in the spatial ultimatum game, *EPL* **100** (2012) 28005.

[2] A. Szolnoki, M. Perc, G. Szabo, Defense mechanisms of empathetic players in the spatial ultimatum game, *Physical Review Letters* **109** (2012) 078701.

Quantum resonances and non-Hermiticity

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The proper description of quantum resonances poses intricate problems in particle physics, especially whenever a relativistic treatment is required. Sometimes even the definition of a quantum resonance state is not unequivocal, in particular from the point of view of experiment and theory.

In standard quantum mechanics one deals with Hermitian Hamiltonians that allow only for discrete and continuous spectra with real eigenvalues. Whereas bound states correspond to poles of the resolvent on the negative real axis of the complex energy plane, resonances are observed as peaks/bumps in the scattering cross section. For the latter one usually gets hold of the resonance parameters (energies and decay widths) by ad-hoc parameterizations.

Considering the analytic structure of the resolvent (or S-operator) as a function of momentum/energy, resonant states are related to poles in the complex momentum/energy planes. Such a description, however, is in principle foreign to a Hermitian theory in Hilbert space. Contrary to bound-state poles the residua at resonance poles (Gamow functions) are generally not square-integrable. Thus many notions of standard quantum theory are lost and a number of practical methods can no longer be applied. Such situations arise frequently, e.g., when complex optical potentials for composite systems are dealt with, resulting from (Feshbach) elimination of open coupled channels.

I discuss the essential modifications that are necessary to formulate a quantum theory for non-Hermitian Hamiltonians, much in the spirit of ref. [1]. Regarding the solvability of such Hamiltonians, I present a modified variational method that makes use of an optimization principle, applicable also for calculating complex eigenenergies, and I demonstrate its efficiency by several numerical examples.

References

[1] N. Moiseyev *Non-Hermitian Quantum Mechanics* (Cambridge University Press, Cambridge, 2011)

Efekti nereda in "antifragilnost" v kulonskih tekočinah

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Predstavil bom konceptualen in formalen okvir za razumevanje kako prisotnost zamrznjenega nereda vpliva na sile med nabitimi makromolekularnimi površinami. Obravnaval bom primere monopolnega oziroma dipolnega površinskega naboja v primeru kulonskih tekočin in pokazal, da prisotnost nereda lahko pripelje do anomalno dolgodosežnih longitudinalnih in lateralnih sil kakor tudi navorov. Te sile, ki jih povzroča nered, ne izginejo niti v primeru nevtralnih površin in otežkočajo razumevanje eksperimentalno opaženih sil, kot so recimo Casimirjeve sile. Pokazal bom tudi, da v primeru močno sklopljenih kulonskih tekočin, ki sestojijo iz mešanice mnogovalentnih in enovalentni soli, lahko pride do zanimivega primera "antifragilnosti", kjer dodaten zamrznjen nered v sistemu dejansko zmanjša termični nered, in omogoča koristno delovanje zunanjega nereda na sistem. Opisano delo je povzel AIP v sporočilu za tisk z naslovom "Nered + nered = še več nereda?", ki so ga kasneje povzele tudi druge novičarske znanstvene agencije ter celo Financial Times v posebnem članku.

Reference

[1] Ali Naji, Matej Kanduč, Jan Forsman, Rudolf Podgornik, Perspective: Coulomb fluids – weak coupling, strong coupling, in between and beyond, *J. Chem. Phys.* **139** (2013) 150901.

[2] Ali Naji, Malihe Ghodrat, Haniyeh Komaie-Moghaddam, Rudolf Podgornik, Asymmetric Coulomb fluids at randomly charged dielectric interfaces: Antifragility, overcharging and charge inversion, *J. Chem. Phys.* **141** (2014) 174704.

Disorder effects and antifragility in Coulomb fluids

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I will present a conceptual and formal framework that allows us to understand how forces between charged macromolecular surfaces are influenced by the presence of quenched disordered charges on the boundaries. I will consider examples of monopolar and/or dipolar surface charge disorder in the case of confined Coulomb fluids and argue, that the presence of disorder can give rise to anomalously long-ranged normal and lateral interaction forces as well as torques. These disorder-generated interactions persist even for overall charge-neutral surfaces and can complicate the disentanglement of other long-range interactions, e.g. the Casimir interaction, from the observed forces. Furthermore, I will argue that in the case of strongly coupled Coulomb fluids, composed of multivalent and monovalent salt mixture with quenched disordered surface charges one can observe a peculiar type of "antifragility" where more imposed disorder actually diminishes the thermal disorder in the system, allowing the system to thrive on disorder. The work described was featured in an AIP press release entitled "Disorder + Disorder = More Disorder?" that was later quoted by many science news sources (sciencedaily, scikon, esciencenews, ...) and even by an article in the Financial Times.

References

[1] Ali Naji, Matej Kanduč, Jan Forsman, Rudolf Podgornik, Perspective: Coulomb fluids – weak coupling, strong coupling, in between and beyond, *J. Chem. Phys.* **139** (2013) 150901.

[2] Ali Naji, Malihe Ghodrat, Haniyeh Komaie-Moghaddam, Rudolf Podgornik, Asymmetric Coulomb fluids at randomly charged dielectric interfaces: Antifragility, overcharging and charge inversion, *J. Chem. Phys.* **141** (2014) 174704.

Iskanje majorana nevtrinov v mezonskih razpadih s kršitvijo leptonskega števila

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Z otkritjem neutrinskih oscilacij je ugotovljeno, da nevtrini imajo maso, vendar je narava te mase je še vedno uganka. Najbolj priljubljena razlaga nevtrinske mase je preko "see-saw" mehanizma, ki predpostavlja, da ob nevtrinih, ki jih predvideva Standardni model, obstajao še dodatni Majorana nevtrini z zelo visokimi masami, ki se zelo šibko mešajo z leptonskim sektorjem Standardnega modela. V zadnjem času pa so se pojavile različice "see-saw" mehanizma, ki napovedujejo, da bi ti dodatni nevtrini lahko imeli mase okrog 1 GeV. Takšne napovedi so zelo zanimive, saj je namreč iskanje teh nevtrinov pri takšnem scenariju mogoče z že obstoječimi ekperimentalnimi postavitvami. Tukaj bom predstavila možnosti za iskanje Majorana nevtrinov v mezonskih razpadih z dvema leptonoma z enakim nabojem v končnem stanju v trkalnikih kot so BaBar, Belle in LHCb.

Reference

[1] A. Atre, T. Han, S. Pascoli and B. Zhang, *JHEP* **0905** (2009) 030.
[2] C. Dib and C. S. Kim, *Phys. Rev. D* **89** (2014) 077301.
[3] G. Cvetic, C. Dib, C. S. Kim and J. Zamora-Saa, *Symmetry* **7** (2015) 726
[4] R. Aaij *et al.* [LHCb Collaboration], *Phys. Lett.* B **724** (2013) 203.
[5] R. Aaij *et al.* [LHCb Collaboration], *Phys. Rev. D* **85** (2012) 112004.
[6] D. Liventsev *et al.* [Belle Collaboration], *Phys. Rev. D* **87** (2013) no.7, 071102.

Searches for Majorana Neutrinos in Lepton Violating Meson Decays

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Although the neutrino oscillation experiments have proven that the neutrinos are massive, the nature of their masses is still unknown. As they are without charge, they can be either Majorana or Dirac particles. According to see-saw mechanism, besides three light neutrinos that are known in the Standard Model, there should exist additional heavy Majorana neutrinos with very small mixing coefficients with SM leptons. Although in the original version of the see-saw mechanism, these so called sterile neutrinos are predicted to have masses much larger than 1 TeV, there are also see-saw theories that allow for sterile neutrino with masses near 1 GeV scale and relatively large mixing coefficients with SM leptons. These predictions are rather interesting as, in such scenarios, one can search for Majorana neutrinos in already existing experiments such as BaBar, Belle and LHCb. Here, I will present the searches for Majorana neutrinos in meson decays with two same charged leptons in the final state.

Reference

[1] A. Atre, T. Han, S. Pascoli and B. Zhang, *JHEP* **0905** (2009) 030.
[2] C. Dib and C. S. Kim, *Phys. Rev. D* **89** (2014) 077301.
[3] G. Cvetic, C. Dib, C. S. Kim and J. Zamora-Saa, *Symmetry* **7** (2015) 726
[4] R. Aaij *et al.* [LHCb Collaboration], *Phys. Lett.* B **724** (2013) 203.
[5] R. Aaij *et al.* [LHCb Collaboration], *Phys. Rev. D* **85** (2012) 112004.
[6] D. Liventsev *et al.* [Belle Collaboration], *Phys. Rev. D* **87** (2013) no.7, 071102.

Od neurejenih spinskih verig do večdelčne lokalizacije

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Hkratno delovanje nereda in elektronskih interakcij je v zadnjem desetletju eden osrednjih teoretičnih izzivov v fiziki kondenzirane snovi. To vprašanje je zanimivo tudi v povezavi z eksperimenti na realnih materialih in pri hladnih atomih na optičnih mrežah. V svojem predavanju bom najprej opisal eksperimente na snoveh z neurejenimi spinskimi verigami, ki nakazujejo anomalno dinamiko. Podal bom tudi teoretično razlago v okviru neurejenega Heisenbergovega modela. Nov pojem predstavlja večdelčna lokalizacija, ki predvideva neergodično obnašanje in odsotnost termalizacije v neurejenih sistemih kljub elektronski interakciji. Ta pojav je, kot kaže, realiziran v eksperimentih na hladnih atomih. Predstavil bom numerične rezultate v okviru standardnega modela večdelčne lokalizacije, ki potrjujejo obstoj takega stanja. Bom pa nakazal tudi nekatere pomembne omejitve tega scenarija.

From disordered spin chains to many-body localization

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The interplay of disorder and electron interactions is in recent decade one of the most challenging theoretical problems in condensed-matter physics. It is interesting also in connection with experiments in real materials and cold gases on optical lattices. In the talk I will discuss first the experiments on materials with random spin chains, which exhibit anomalous dynamics, as well the theoretical explanation within the framework of random Heisenberg model. New paradigm is many-body localization, which predicts in systems with strong disorder the nonergodicity and absence of thermalization even in the presence of interaction, and appears to be realized in cold-atom systems, I will present some recent numerical results within the standard model of many-body localization, which confirm the existence od such a state, but I will show also limitations of this phenomenon.

Močno korelirana neravnovesna stacionarna stanja s tokom — klasična in kvantna slika

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V predavanju bom orisal nekaj preprostih eksplicitnih modelov močno koreliranih stacionarnih stanj neravnovesnih konzervativnih sistemov v eni dimenziji, ki jih iz ravnovesja poganja disipativna sklopitev z okolico na robovih sistema. Vse odlikuje preprosta algebrajska struktura točne rešitve, ki jo ponazarja matrično produktni nastavek. V okviru kvantne fizike so to npr. integrabilne spinske verige, kot je npr. XXZ model, ali Fermi-Hubbardov model [1], v okviru klasične mehanike pa je zanimiv primer takšnega modela reverzibilni integrabilni celični avtomat [2]. Predstavil bom splošne orise metode točnega reševanja neravnovesnih stacionarnih stanj z matrično-produktnim nastavkom in njegovimi posplošitvami in nekatera najbolj zanimiva odprta vprašanja.

Reference

[1] T. Prosen, *Topical review:* Matrix product solutions of boundary driven quantum chains, J. Phys. A: Math. Theor. **48**, 373001 (2015).

[2] T. Prosen in C. Mejia-Monasterio, Integrability of a deterministic cellular automaton driven by stochastic boundaries, J. Phys. A: Math. Theor. **49**, 185003 (2016).

Strongly correlated nonequilibrium steady states with currents — classical and quantum picture

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In my talk I will introduce several explicit models of strongly correlated stationary states of conservative systems in one dimension that are driven out of equilibrium with the dissipative couplings at the system boundaries. All these models share a simple algebraic matrix product structure of the exact solution. In the framework of quantum physics, the main examples of such models are integrable spin chains, e.g. the XXZ model, or Fermi-Hubbard model [1], while in the realm of classical physics we have an example of a reversible and integrable cellular automaton [2]. I will outline general features of solving nonequilibrium stationary states in terms of matrix product ansatz and its generalizations and stress some of the most interesting and outstanding open problems.

Reference

[1] T. Prosen, *Topical review:* Matrix product solutions of boundary driven quantum chains, J. Phys. A: Math. Theor. **48**, 373001 (2015).

[2] T. Prosen and C. Mejia-Monasterio, Integrability of a deterministic cellular automaton driven by stochastic boundaries, J. Phys. A: Math. Theor. **49**, 185003 (2016).

Kvantni kaos v generičnih sistemih

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Kvantni kaos (ali valovni kaos) je področje v teoretični in eksperimentalni fiziki, ki se ukvarja s pojavi v kvantni domeni (še posebej kar zadeva rešitve Schrödingerjeve enačbe), ali v drugih valovnih sistemih, ki zrcalijo klasični kaos. Ti drugi valovni sistemi so elektromagnetno valovanje, akustično, elastično valovanje, površinki valovi, seizmični, gravitacijski valovi, itd. Klasična dinamika opisuje "žarke" danih valov, most med klasično in kvantno mehaniko pa je semiklasična mehanika, ki je osnovana na kratko-valovnih aproksimacijah. Če je klasična dinamika kaotična, jasno vidimo njene sledove v kvantni (valovni) domeni, n.pr. v statističnih lastnostih diskretnega energijskega spektra, v strukturi lastnih funkcij, in v statističnih lastnostih drugih opazljivk. Kvantni kaos se pojavlja v nizko-dimenzionalnih sistemih, n.pr. s samo dvema prostostnima stopnjama (n.pr. v 2D biljardih), pa tudi v multidimenzionalnih sistemih. Iz navedenega je očitno, da so teorija in eksperimenti v kvantnem kaosu fundamentalnega pomena v fiziki, in še več, tudi v tehnologiji.

V generičnih Hamiltonovih sistemih imamo območja stabilnega, regularnega, gibanja v klasičnem faznem prostoru za določene začetne pogoje, in kaotična območja za komplementarne začetne pogoje. Temu ustrezno so pripadajoča lastna stanja bodisi regularna ali kaotična, in tudi pripadajoči energijski spektri imajo različno statistiko, namreč bodisi Poissonovo za regularna stanja ali pa statistiko naključnih matrik za kaotična stanja. Da bi lahko odločili, ali so dana stanja in pripadajoči energijski spektri regularni ali kaotični, moramo gledati strukturo Wignerjevih funkcij v "kvantnem faznem prostoru".

Kvantna lokalizacija klasično kaotičnih lastnih stanj je ena od najpomembnejših pojavov v kvantnem kaosu, ali bolj splošno - valovnem kaosu, skupaj s karakterističnim vedenjem statističnih lastnosti energijskih spektrov. Kvantna lokalizacija se pojavi, ko je Heisenbergov čas t_H danega sistema krajši kot klasični transportni čas danega klasičnega sistema, t.j. kadar je klasični transport počasnejši kot kvantni čas resolucije evolucijskega operatorja. Heisenbergov čas, kot pomembna karakterizacija vsakega kvantnega sistema, je namreč enak razmerju Planckove konstante $2\pi\hbar$ in

srednjega razmika med sosednjimi energijskimi nivoji ΔE, $t_H = 2\pi\hbar/\Delta E$.

Pokazali bomo funkcionalno odvisnost med stopnjo lokalizacije in spektralno statistiko v avtonomnih (časovno neodvisnih) sistemih, v analogiji z brcanim kvantnim rotatorjem, ki je paradigma časovno periodičnih (Floquetovih) sistemov, in nadalje predstavili pristop in metodo na primeru biljardnega sistema v dinamičnem režimu med integrabilnostjo (krog) ter polnim kaosom (kardioidni biljard), kjer bomo ekstrahirali kaotična stanja. Stopnja lokalizacije je določena z dvema različnima lokalizacijskima merama, uporabljajoč Poincaré-Husimijeve funkcije (ki so gaussovsko glajene Wignerjeve funkcije v faznem prostoru Poincaréja-Birkhoffa), ki so pozitivno definitne in jih lahko obravnavamo kot kvazi-verjetnostne gostote. Prva mera A je definirana s pomočjo informacijske entropije, medtem ko je druga, C, definirana s pomočjo korelacij v faznem prostoru Poincaré-Husimijevih funkcij lastnih stanj. Presenetljivo, in zadovoljivo, izkaže se, da sta obe meri premo sorazmerni in tako ekvivalentni.

Ena od glavnih manifestacij kaosa v kaotičnih lastnih stanjih, ko ni kvantne lokalizacije, je porazdelitev razmikov med sosednjimi nivoji $P(S)$, ki je pri majhnih S linearna $P(S) \propto S$, in govorimo o linearnem odbijanju med sosednjimi nivoji, medtem ko imamo v integrabilnih sistemih Poissonovo statistiko (eksponentno funkcijo $P(S) = \exp(-S)$), kjer ni odbijanja med sosednjimi nivoji ($P(0) = 1 \neq 0$). V povsem kaotičnem režimu s kvantno lokalizacijo pa opazimo, da je $P(S)$ pri majhnih S potenčna funkcija $P(S) \propto S^{\beta}$, z $0 < \beta < 1$. Pokazali bomo, da obstaja funkcionalna odvisnost med lokalizacijsko mero A in eksponentom β, namreč da je β monotona funkcija A: v primeru močne lokalizacije sta A in β majhna, blizu 0, medtem ko sta v primeru šibke lokalizacije (skoraj razširjena kaotična stanja) A in β blizu 1.

Reference

[1] F. Haake, *Quantum Signatures of Chaos* (Berlin: Springer, 2001).

[2] H.-J. Stöckmann, *Quantum Chaos: An Introduction* (Cambridge University Press, 1999).

[3] B. Batistić and M. Robnik, *Phys.Rev. E* **88** (2013) 052913.

[4] M.V. Berry and M. Robnik, *J. Phys.A: Math.& Gen.* **17** (1984) 2413.

[5] B. Batistić and M. Robnik, *Journal of Physics A: Mathematical & Theoretical* **43** (2010) 215101.

[6] B. Batistić and M. Robnik, *Journal of Physics A: Mathematical & Theoretical* **45** (2013) 315102.

[7] B. Batistić, T. Manos and M. Robnik, *Europhys. Lett.* **102** (2013) 50008.

[8] M. Robnik, *Nonlinear Phenomena in Complex Systems* (Minsk) **1** (1998) No. 1, 22.

[9] T. Manos and M. Robnik, *Phys. Rev. E* **87** (2013) 062905.

[10] M. Robnik, *J. Phys.A: Math.& Gen..* **16** (1983) 3971.

[11] M. Robnik, *J. Phys.A: Math.& Gen..* **17** (1984) 1049.

[12] T. Prosen and M. Robnik, *J. Phys.A: Math.& Gen..* **27** (1994) 8059.

[13] T. Prosen and M. Robnik, *J. Phys.A: Math.& Gen..* **32** (1999) 1863.

[14] T. Manos and M. Robnik, *Physical Rev. E* **91** (2015) 042904-1.

[15] T. Manos and M. Robnik, *Nonlinear phenomena in complex systems* **18** (2015) No. 3, 335.

[16] M. Robnik, *The European Physical Journal Special Topics* **225** iss. 6/7 (2016) 959.

Quantum chaos of generic systems

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Quantum chaos (or wave chaos) is a research field in theoretical and experimental physics dealing with the phenomena in the quantum domain (especially regarding solutions of the Schroedinger equation), or in other wave systems, which correspond to the classical chaos. These other wave systems are electromagnetic, acoustic, elastic, surface, seismic, gravitational waves, etc. The classical dynamics describes the "rays" of the underlying waves, and the bridge between the classical and quantum mechanics is the semiclassical mechanics, resting upon the short-wavelength approximations. If the classical dynamics is chaotic, we see clear signatures in the quantum (wave) domain, e.g. in statistical properties of discrete energy spectra, in the structure of eigenfunctions, and in the statistical properties of other observables. Quantum chaos occurs in low-dimensional systems, e.g. with just two degrees of freedom (e.g. in 2D billiards), but of course also in multi-dimensional systems. From the above it is obvious that theory and experiment in quantum chaos are of fundamental importance in physics, and, moreover, also in technology.

In generic Hamilton systems we have regions of stable, regular, motion in the classical phase space for certain initial conditions, and chaotic motion for the complementary initial conditions. Accordingly, the corresponding eigenstates are either regular or chaotic, and also the corresponding energy spectra have different statistical properties, namely either Poissonian for the regular eigenstates or the statistics of random matrices in the chaotic case. In order to decide whether a given eigenstate and the corresponding energy level is regular or chaotic, we must look into the structure of Wigner functions in the "quantum phase space".

Quantum localization of classically chaotic eigenstates is one of the most important phenomena in quantum chaos, or more generally - wave chaos, along with the characteristic behaviour of statistical properties of the energy spectra. Quantum localization sets in, if the Heisenberg time t_H of the given system is shorter than the classical transport times of the underlying classical system, i.e. when the classical transport is slower than the quantum time resolution of the evolution operator. The

Heisenberg time t_H, as an important characterization of every quantum system, is namely equal to the ratio of the Planck constant $2\pi\hbar$ and the mean spacing between two nearest energy levels ΔE, $t_H = 2\pi\hbar/\Delta E$.

We shall show the functional dependence between the degree of localization and the spectral statistics in autonomous (time independent) systems, in analogy with the kicked rotator, which is the paradigm of the time periodic (Floquet) systems, and shall demonstrate the approach and the method in the case of a billiard family in the dynamical regime between the integrability (circle) and full chaos (cardioid), where we shall extract the chaotic eigenstates. The degree of localization is determined by two localization measures, using the Poincaré Husimi functions (which are the Gaussian smoothed Wigner functions in the Poincaré Birkhoff phase space), which are positive definite and can be treated as quasi-probability densities. The first measure A is defined by means of the information entropy, whilst the second one, C, in terms of the correlations in the phase space of the Poincaré Husimi functions of the eigenstates. Surprisingly, and very satisfactory, the two measueres are linearly related and thus equivalent.

One of the main manifestations of chaos in chaotic eigenstates in absence of the quantum localization is the energy level spacing distribution $P(S)$ (of nearest neighbours), which at small S is linear $P(S) \propto S$, and we speak of the linear level repulsion, while in the integrable systems we have the Poisson statistics (exponential function $P(S) = \exp(-S)$), where there is no level repulsion ($P(0) = 1 \neq 0$). In fully chaotic regime with quantum localization we observe that $P(S)$ at small S is a power law $P(S) \propto S^{\beta}$, with $0 < \beta < 1$. We shall show that there is a functional dependence between the localization measure A and the exponent β, namely that β is a monotonic function of A: in the case of the strong localization are A and β small, while in the case of weak localization (almost extended chaotic states) A and β are close to 1.

References

[1] F. Haake, *Quantum Signatures of Chaos* (Berlin: Springer, 2001).

[2] H.-J. Stöckmann, *Quantum Chaos: An Introduction* (Cambridge University Press, 1999).

[3] B. Batistić and M. Robnik, *Phys.Rev. E* **88** (2013) 052913.

[4] M.V. Berry and M. Robnik, *J. Phys.A: Math.& Gen.* **17** (1984) 2413.

[5] B. Batistić and M. Robnik, *Journal of Physics A: Mathematical & Theoretical* **43** (2010) 215101.

[6] B. Batistić and M. Robnik, *Journal of Physics A: Mathematical & Theoretical* **45** (2013) 315102.

[7] B. Batistić, T. Manos and M. Robnik, *Europhys. Lett.* **102** (2013) 50008.

[8] M. Robnik, *Nonlinear Phenomena in Complex Systems* (Minsk) **1** (1998) No. 1, 22.

[9] T. Manos and M. Robnik, *Phys. Rev. E* **87** (2013) 062905.

[10] M. Robnik, *J. Phys.A: Math.& Gen..* **16** (1983) 3971.

[11] M. Robnik, *J. Phys.A: Math.& Gen..* **17** (1984) 1049.

[12] T. Prosen and M. Robnik, *J. Phys.A: Math.& Gen..* **27** (1994) 8059.

[13] T. Prosen and M. Robnik, *J. Phys.A: Math.& Gen..* **32** (1999) 1863.

[14] T. Manos and M. Robnik, *Physical Rev. E* **91** (2015) 042904-1.

[15] T. Manos and M. Robnik, *Nonlinear phenomena in complex systems* **18** (2015) No. 3, 335.

[16] M. Robnik, *The European Physical Journal Special Topics* **225** iss. 6/7 (2016) 959.

Prvi integrali May-Leonardevega asimetričnega sistema

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Obravnavamo problem obstoja prvih integralov v tri-dimenzionalnem sistemu May-Leonarda. Z uporabo sistemov računske algebre Mathematica in Singular smo našli poddružine May-Leonardevega asimetričnega sistema z invariantnimi ravninami in invariantnimi ploskvami drugega reda. Te ploskve smo uporabili za določitev poddružin May-Leonardevega sistema, ki imajo analitični prvi integral.

Reference

[1] V. Antonov, D. Dolićanin, V.G. Romanovski, J. Tóth, *MATCH Commun. Math. Comput. Chem.* **76** (2016) 455–474.

[2] V. Antonov, W. Fernandes, V.G. Romanovski, N.L. Shcheglova, First integrals of the May-Leonard asymmetric system, 2016, submitted.

First integrals of the May-Leonard asymmetric system

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We investigate existence of first integrals in the three dimensional May-Leonard asymmetric system. Using the computational algebra systems Mathematica and Singular we first look for subfamilies of the May-Leonard asymmetric system admitting invariant surfaces of degree one and two. Then using these invariant surfaces we identify subfamilies of the system admitting analytic first integral.

References

[1] V. Antonov, D. Dolićanin, V.G. Romanovski, J. Tóth, *MATCH Commun. Math. Comput. Chem.* **76** (2016) 455–474.

[2] V. Antonov, W. Fernandes, V.G. Romanovski, N.L. Shcheglova, First integrals of the May-Leonard asymmetric system, 2016, submitted.

Isochronicity and linearizability of a planar cubic system

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In this talk we investigate the problem of isochronous center for a cubic complex planar system of ordinary differential equations whose infinity is degenerate and it is full of equilibria. Chavarriga, Giné and García, [*Differ. Equ. Dynam. Syst.* **7**, (1999) 221-238] have found five families in polar coordinates of the real system having an isochronous center at the origin. We make a complete classification of isochronous systems in the family obtaining the necessary and sufficient conditions on parameters of system for the existence of isochronous center when the parameters are complex. A flaw in the paper mentioned above is corrected. At last, we discuss the coexistence of isochronous centers for the system. It is a joint work with W. Fernandes, V.G. Romanovski and M.S. Sultanova.

References

[1] J. Chavarriga, J. Giné and I. García, *Differ. Equ. Dynam. Syst.* **7** (1999) 221-238.
[2] M. Han, V.G. Romanovski and X. Zhang, *Rom. Journ. Phys.* **61** (2016) 157-166.
[3] N. G. Lloyd, C. J. Christopher, J. Devlin, J. M. Pearson and N. Yasmin, *Differential Equations and Dynamical Systems* **5** (1997) 329-345.
[4] V. G. Romanovski and M. Robnik, *J. Phys. A, Math. Theor.* **34** (2001) 10267-10292.
[5] V. G. Romanovski and D. S. Shafer, *The Center and cyclicity Problems: A computational Algebra Approach*, Boston: Birkhauser, 2009.

Samo-organizirana kritičnost kot princip organizacije v populacijah celic beta v pankreasu

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V človeškem telesu je približno milijarda celic beta. Najdemo jih v približno milijon mikroorganih, razporejenih po trebušni slinavki, ki se imenujejo Langerhansovi otočki in od katerih vsak vsebuje približno tisoč celic beta, ki izločajo anabolni hormon inzulin in igrajo glavno vlogo v uravnavanju shranjevanja in porabe energijsko bogatih molekul, kot je recimo glukoza (Dolenšek, Rupnik *et al.* 2015). Koncentracija tako inzulina kot glukoze v krvi oscilira, in sicer najverjetneje zaradi pulzatilnega izločanja inzulina iz trebušne slinavke, ki je mogoč na račun močne sklopitve med celicami beta znotraj otočkov in šibke sklopitve med posameznimi otočki (Satin, Butler *et al.* 2015). V vsaki celici beta so izražene oscilacije znotrajcelične koncentracije kalcijevih ionov ($[Ca^{2+}]_{IC}$), ki poganjajo pulze izločanja inzulina in te oscilacije so med različnimi celicami beta znotraj otočka poravnane na račun valov $[Ca^{2+}]_{IC}$ (Dolenšek, Stožer *et al.* 2013; Stožer, Dolenšek *et al.* 2013). Oscilacije in valovi $[Ca^{2+}]_{IC}$ pa po svoji naravi nikakor niso trivialni in sklopitev med celicami je zelo heterogena, zaradi česar je zelo težavno spremljati aktivnost celic beta in razumeti organizacijske principe, ki narekujejo njihovo obnašanje (Stožer, Gosak *et al.* 2013; Gosak, Stožer *et al.* 2015; Markovič, Stožer *et al.* 2015). V pričujoči študiji smo se tega problema lotili v dveh korakih. V prvem smo zgradili računski model heterogenih in heterogeno sklopljenih celic beta in statistično analizirali prostorsko-časovno organizacijo velikosti medceličnih valov $[Ca^{2+}]_{IC}$. Obnašanje celic beta smo simulirali za konstantno stimulacijo z glukozo, kot tudi za oscilatorno stimulacijo, pri čemer je slednja bližje fiziološkim

razmeram, pri katerih koncentraciji inzulina in glukoze oscilirata. V drugem koraku smo primerjali napovedi modela z eksperimentalnimi podatki, pridobljenimi s pomočjo konfokalnega mikroskopskega snemanja oscilacij in valov $[Ca^{2+}]_{IC}$ v celicah beta v svežih tkivnih rezinah, ki smo jih izpostavili enakim stimulacijskim protokolom kot v simulacijah. Tako simulacije kot eksperimenti so pokazali, da so spremembe $[Ca^{2+}]_{IC}$ v celicah beta v primeru konstantne stimulacije z glukozo dvofazne, z aktivacijsko in platojsko fazo. Med aktivacijsko fazo, ki traja prvih 5 minut za začetkom stimulacije z glukozo, se celice beta rekrutirajo in statistična razporeditev velikosti kalcijevih dogodkov sledi potenčni, kar nakazuje na kritično obnašanje. Po začetni aktivacijski fazi se dinamična narava kvalitativno spremeni in postane bolj stabilna in organizirana. V tem t.i. platojskem režimu se pojavi več velikih, to je globalnih dogodkov, kar nakazuje na to, da je obnašanje v tem režimu superkritično. Bolj fiziološka intermitentna stimulacija je vodila v odziv, ki je bil po svoji naravi vedno samo kritičen, tudi po daljšem času stimulacije. Naši rezultati ponujajo novo razlago za evolucijsko vlogo oscilatorne narave izločanja inzulina in nakazujejo na možen dodaten mehanizem za disfunkcijo celic beta med sladkorno boleznijo, pri kateri je oscilatoren vzorec spreminjanja koncentracije inzulina in glukoze porušen in zato morda pride do premika iz kritičnega v superkritični režim obnašanja.

Reference

[1] Dolenšek, J., M. S. Rupnik, et al. (2015). "Structural similarities and differences between the human and the mouse pancreas." Islets 7(1): e1024405.

[2] Dolenšek, J., A. Stožer, et al. (2013). "The Relationship between Membrane Potential and Calcium Dynamics in Glucose-Stimulated Beta Cell Syncytium in Acute Mouse Pancreas Tissue Slices." PLoS ONE 8(12): e82374.

[3] Gosak, M., A. Stožer, et al. (2015). "The relationship between node degree and dissipation rate in networks of diffusively coupled oscillators and its significance for pancreatic beta cells." Chaos 25(7): 073115.

[4] Markovič, R., A. Stožer, et al. (2015). "Progressive glucose stimulation of islet beta cells reveals a transition from segregated to integrated modular functional connectivity patterns." Sci. Rep. 5: 7845

[5] Satin, L. S., P. C. Butler, et al. (2015). "Pulsatile insulin secretion, impaired glucose tolerance and type 2 diabetes." Mol Aspects Med. 42: 61

[6] Stožer, A., J. Dolenšek, et al. (2013). "Glucose-Stimulated Calcium Dynamics in Islets of Langerhans in Acute Mouse Pancreas Tissue Slices." PLoS ONE 8: e54638.

[7] Stožer, A., M. Gosak, et al. (2013). "Functional Connectivity in Islets of Langerhans from Mouse Pancreas Tissue Slices." PLoS Comput Biol 9(2): e1002923.

Self-organized criticality as an organizing principle in pancreatic beta cell populations

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There are about a billion beta cells in the human body. They can be found in about a million microorgans dispersed within the pancreas and called islets of Langerhans, each of which contains about a thousand beta cells that secrete the anabolic hormone insulin and play the main role in regulating storage and usage of energy-rich nutrients, such as glucose (Dolenšek, Rupnik *et al.* 2015). The levels of both insulin and glucose oscillate in blood and this is probably due to pulsatile release of insulin from the pancreas, which is possible due to strong intercellular coupling between beta cells within islets and weak coupling between islets (Satin, Butler *et al.* 2015). Each beta cell displays oscillations of intracellular calcium concentration ($[Ca^{2+}]_{IC}$) that drive pulses of insulin secretion and these oscillations are aligned between different cells within an islet by means of $[Ca^{2+}]_{IC}$ waves (Dolenšek, Stožer *et al.* 2013; Stožer, Dolenšek *et al.* 2013). However, the $[Ca^{2+}]_{IC}$ oscillations and waves are nontrivial and the coupling between cells seems to be very heterogeneous and it is therefore extremely difficult to track the activity of beta cells and understand the organizing principles governing their behavior (Stožer, Gosak *et al.* 2013; Gosak, Stožer *et al.* 2015; Markovič, Stožer *et al.* 2015). In the present study, we approached this problem in two steps. First, we constructed a computational model of heterogeneous and heterogeneously coupled beta cells and statistically analyzed the spatio-temporal organization of intercellular $[Ca^{2+}]_{IC}$ wave sizes. Beta cell behavior was simulated for a constant stimulation with glucose, as well as for an oscillatory stimulation, the latter more closely mimicking the physiologic conditions with

oscillating blood glucose and insulin levels. In the second step, we compared the model predictions with experimental data obtained by means of confocal imaging of $[Ca^{2+}]_{IC}$ oscillations and waves in beta cells in acute tissue slices subjected to the same stimulation protocols as in simulations. Both computational and experimental results showed that the $[Ca^{2+}]_{IC}$ responses of beta cells after constant stimulation with glucose are characterized by a two-phased dynamics: an activation phase and a plateau phase. During the activation phase that encompasses the initial 5 minutes after stimulation with glucose, the beta cells are being recruited and the distribution of $[Ca^{2+}]_{IC}$ event sizes is characterized by a power law distribution, suggesting a critical behavior. After this initial critical phase, the dynamical nature changes qualitatively and becomes more stable and organized. In this plateau regime, a high number of bigger,*i.e.*, more global $[Ca^{2+}]_{IC}$ events are observed, indicating that the $[Ca^{2+}]_{IC}$ dynamics during the plateau phase is super-critical. The more physiological intermittent stimulation with glucose led to a behavior that was critical only, even after long periods of time. Our results suggest a novel evolutionary role for the oscillatory nature of insulin secretion and suggest an additional mechanism for beta cell dysfunction in diabetes where the oscillatory pattern of plasma insulin and glucose changes is disrupted, possibly shifting the beta cell behavior from critical to supercritical.

References

[1] Dolenšek, J., M. S. Rupnik, et al. (2015). "Structural similarities and differences between the human and the mouse pancreas." Islets 7(1): e1024405.

[2] Dolenšek, J., A. Stožer, et al. (2013). "The Relationship between Membrane Potential and Calcium Dynamics in Glucose-Stimulated Beta Cell Syncytium in Acute Mouse Pancreas Tissue Slices." PLoS ONE 8(12): e82374.

[3] Gosak, M., A. Stožer, et al. (2015). "The relationship between node degree and dissipation rate in networks of diffusively coupled oscillators and its significance for pancreatic beta cells." Chaos 25(7): 073115.

[4] Markovič, R., A. Stožer, et al. (2015). "Progressive glucose stimulation of islet beta cells reveals a transition from segregated to integrated modular functional connectivity patterns." Sci. Rep. 5: 7845

[5] Satin, L. S., P. C. Butler, et al. (2015). "Pulsatile insulin secretion, impaired glucose tolerance and type 2 diabetes." Mol Aspects Med. 42: 61

[6] Stožer, A., J. Dolenšek, et al. (2013). "Glucose-Stimulated Calcium Dynamics in Islets of Langerhans in Acute Mouse Pancreas Tissue Slices." PLoS ONE 8: e54638.

[7] Stožer, A., M. Gosak, et al. (2013). "Functional Connectivity in Islets of Langerhans from Mouse Pancreas Tissue Slices." PLoS Comput Biol 9(2): e1002923.

Reševanje izbranih problemov v arheometriji z metodo najmanjšega raztrosa rezultatov

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Pri nekaterih merskih problemih ne moremo enolično določiti rešitve, ker ne poznamo določenih fizikalnih parametrov. V tem primeru si lahko pomagamo z množico meritev, ki jih opravimo pri rahlo spremenjenih začetnih pogojih in nato poiščemo rešitev z metodo najmanjĕga raztrosa končnih rezultatov. Opisali bomo dva arheometrična problema. Prvi zadeva datiranje arheološke malte z metodo C-14. Težava pri tovrstnem datiranju je mešanje arheološkega in geološkega apnenca, ki povzroči razredčenje koncentracije C-14 in tako vodi do prevelikih starosti. Predlagamo ekstrapolacijski postopek, ki temelji na časovnem spreminjanju izotopa C-13. Začetne koncentracije C-13 obravnavamo kot variacijski parameter, s katerim dobimo najmanjše razsipanje dobljenih starosti. Drugi problem zadeva balistiko sestavljenih puščic iz trstike, s katerimi posnemamo zgodovinske zglede. V strelskih poskusih smo poskusili določiti začetno hitrost puščice, tako da smo merili njen domet in čas leta. Pri tem je neznani parameter presek puščice, pomnožen s koeficientom upora. Začetno hitrost izstrelka določimo iz skupine strelov pod različnimi koti, tako da je razsipanje začetnih hitrosti minimalno.

Application of regression analysis in selected archaeometrical problems

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Insufficient knowledge of physical parameters sometimes does not allow unique solution of particular measuring problems. In this case, a solution can nevertheless be obtained from repeated measurements performed under slightly varied conditions and minimizing the spread of results by regression analysis. We shall describe two such problem in the field of archaeometry. The first problem concerns dating of ancient mortar by the C-14 method. The main problem in dating mortar is mixing of archeological and mineral limestone, which results in dilution of C-14 content in the sample, thus leading to overestimated ages. We propose an extrapolation scheme based on the time variation of carbon isotope C-13. Initial C-13 concentrations are treated as variation parameters that yield minimum spread of C-14 dates. The second problem concerns ballistics of composite arrows made of reed that simulate historic examples. In shooting experiments, we tried to determine the initial projectile velocity, measuring the range and time of flight. As the cross section of the arrow multiplied by its ballistic coefficient is unknown, the arrow initial velocities were obtained assuming their minimum spread obtained for shots at different take-off angles.

Floquet topological phases coupled to environments and the induced photocurrent

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We consider the fate of a helical edge state of a spin Hall insulator and its topological transition in presence of a circularly polarized light when coupled to various forms of environments. A Lindblad type equation is developed to determine the fermion occupation of the Floquet bands. We find by using analytical and numerical methods that non-secular terms, corresponding to 2-photon transitions, lead to a mixing of the band occupations, hence the light induced photocurrent is in general not perfectly quantized in the presence of finite coupling to the environment, although deviations are small in the adiabatic limit. Sharp crossovers are identified at driving frequencies near the Rabi frequency Ω (which is the strength of light-matter coupling) and at $\frac{1}{2}\Omega$ with the former resembling to a phase transition.

References

[1] B. Dóra, J. Cayssol, F. Simon, and R. Moessner, *Phys. Rev. Lett.* **108** 056602 (2012)

[2] Sz. Vajna, B. Horovitz, B. Dóra, G. Zaránd, *Phys. Rev. B* **94** 115145 (2016)

Differential rings for 2-parameter Calabi-Yau sigma models

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Yamaguchi and Yau showed in [1] that the non-holomorphic content of the special geometry of the mirror quintic moduli space can be phrased in terms of finitely many functions which close under differentiation - the differential ring. This construction was later generalized to arbitrary Calabi-Yau threefolds in [2] and to elliptic curve in [3]. Generators of the differential rings in these cases can be chosen to be quasi-modular forms. It was recently shown that the differential ring can be derived from the algebraic form of the tt^* equations, governing the geometry of the Calabi-Yau moduli space [4]. This allowed the computation of differential rings for Calabi-Yau manifolds with $\dim \leq 3$.

In my talk I will present a systematic derivation of the differential ring from the flat tt^* connection. I will generalize the results of [4] to a particular 2-parameter model and comment on the modularity of the generators of the differential ring.

References

[1] S. Yamaguchi and S. T. Yau, *JHEP* **07** (2004) 047.
[2] M. Alim and J. D. Lange, *JHEP* **0710** (2007) 045.
[3] S. Hosono, *New developments in algebraic geometry, integrable systems and mirror symmetry (RIMS, Kyoto, 2008)* **32** (2010) 79-110.
[4] M. Alim, *arXiv:1412.3454v1 [hep-th]* (2014).

Kvazilokalnost ohranitvenih zakonov modela Hirote

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Model Hirote je integrabilna diskretizacija modela Sinus-Gordon iz teorije polja. Ustreza periodično gnani verigi cikličnih spinov. Predstavil bom izgradnjo kvazilokalnih ohranitvenih zakonov iz Faddeev-Volkovih integralov gibanja.

Reference

[1] L. Zadnik, T. Prosen, *v delu.*

[2] L. Fadjejev, A. J. Volkov, *Letters in Mathematical Physics* **32** (1994) 125-135.

[3] A. J. Volkov, *Physics Letters A* **167** (1992) 345-355.

[4] E. Ilievski, M. Medenjak, T. Prosen, *PRL* **115** (2015) 120601.

[5] A. Antonov, *Theor. Math. Phys.* **113** (1997) 1520-1529.

[6] V. Bažanov, A. Bobenko, N. Rešetikin, *Commun. Math. Phys.* **175** (1996) 377-400.

Quasilocality of conservation laws of the Hirota model

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Hirota model is an integrable discretization of the Sine-Gordon model of the field theory. It corresponds to a periodically driven chain of cyclic spins. I will present the construction of quasilocal conserved charges from Faddeev-Volkov integrals of motion.

References

[1] L. Zadnik, T. Prosen, *in progress.*

[2] L. Faddeev, A. Yu. Volkov, *Letters in Mathematical Physics* **32** (1994) 125-135.

[3] A. Yu. Volkov, *Physics Letters A* **167** (1992) 345-355.

[4] E. Ilievski, M. Medenjak, T. Prosen, *PRL* **115** (2015) 120601.

[5] A. Antonov, *Theor. Math. Phys.* **113** (1997) 1520-1529.

[6] V. Bazhanov, A. Bobenko, N. Reshetikhin, *Commun. Math. Phys.* **175** (1996) 377-400.

Transport in nered

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Čisti sistemi brez nereda in sklopitve z okolico so ponavadi le idealizacija. Zato je pomembno razumeti, kako se lastnosti sistema spremenijo, kadar je prisoten nered. Študiramo lahko razne abstraktne lastnosti, kot so spekter ali pa lastna stanja, ali pa merljive količine, na primer transportne koeficiente. V predavanju bom opisal transportne lastnosti enostavnih sistemov v prisotnosti nereda.

References

[1] R. Nandkishore and D. A. Huse, *Annu. Rev. Condens. Matter Phys.* **6** (2015) 15-38.

[2] M. Žnidarič, A. Scardicchio, and V. K. Varma, *Phys. Rev. Lett.* **117** (2016) 040601.

Transport and disorder

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Clean systems without disorder and external coupling are usually just an idealization. An important question is, how do properties of a system change when disorder is present. One can study various abstract properties, like the spectrum or the eigenfunctions, or, one can consider experimentally measurable quantities like for instance transport. I will discuss transport properties of simple systems in the presence of disorder.

References

[1] R. Nandkishore and D. A. Huse, *Annu. Rev. Condens. Matter Phys.* **6** (2015) 15-38.

[2] M. Žnidarič, A. Scardicchio, and V. K. Varma, *Phys. Rev. Lett.* **117** (2016) 040601.

Dinamika večdelčnih kvantnih sistemov z interakcijo dolgega dosega

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Sistemi z interakcijo dolgega dosega imajo veliko posebnih lastnosti, kot so negativna specifična toplota, neenakost statističnih ansamblov in kvazistacionarna stanja. Po krajši predstavitvi bistvenih razlik med sistemi z interakcijo kratkega in dolgega dosega se bom osredotočil na dinamične lastnosti sistemov z interakcijo dolgega dosega. Obravnaval bom dva pojma dinamične kritičnosti. Prvi je definiran s pomočjo dinamičnega ureditvenega parametra, drugi pa na podlagi neanalitičnosti pri časovnem razvoju določenih nelokalnih opazljivk. Pokazal bom, da temeljita na istem fizikalnem principu.

Reference

[1] Dauxois, T. and Ruffo, S. and Arimondo, E. and Wilkens, M, *Dynamics and thermodynamics of systems with long-range interactions*, Springer, 2002

[2] Markus Heyl, Anatoli Polkovnikov, and Stefan Kehrein, *Phys. Rev. Lett.* **110**, 135704 (2013).

[3] Bojan Žunkovič, Alessandro Silva, and Michele Fabrizio, *Phil. Trans. R. Soc. A* **374**, 20150160 (2016).

[4] B. Žunkovič, M. Heyl, M. Knap, and A. Silva, *arXiv:1609.08482*

[5] B. Neyenhuis, J. Smith, A. C. Lee, J. Zhang, P. Richerme, P. W. Hess, Z. X. Gong, A. V. Gorshkov, and C. Mon- roe, *arXiv:1608.00681.*

Dynamics of quantum many-body systems with long-range interactions

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Systems with long-range interactions exhibit interesting properties such as negative specific heat, non-equivalence of canonical and micro-canonical ensembles and quasi-stationary states. After a short introduction on classical and quantum long-range systems I will focus on dynamical properties of quantum long-range systems. In particular I will relate two notions of dynamical phase transitions. The first is signalled by a non-vanishing time-averaged order parameter and the second is signalled by non-analyticities in the time evolution of specific non-local observables. I will show that they are based on the same physical principle.

References

[1] Dauxois, T. and Ruffo, S. and Arimondo, E. and Wilkens, M, *Dynamics and thermodynamics of systems with long-range interactions*, Springer, 2002

[2] Markus Heyl, Anatoli Polkovnikov, and Stefan Kehrein, *Phys. Rev. Lett.* **110**, 135704 (2013).

[3] Bojan Žunkovič, Alessandro Silva, and Michele Fabrizio, *Phil. Trans. R. Soc. A* **374**, 20150160 (2016).

[4] B. Žunkovič, M. Heyl, M. Knap, and A. Silva, *arXiv:1609.08482*

[5] B. Neyenhuis, J. Smith, A. C. Lee, J. Zhang, P. Richerme, P. W. Hess, Z. X. Gong, A. V. Gorshkov, and C. Mon- roe, *arXiv:1608.00681.*