

# Statistical properties of 1D parametrically kicked Hamilton systems

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I shall discuss the general theory of parametrically kicked systems, especially in non-linear 1D Hamiltonian systems. I shall present the general Papamikos-Robnik (PR) conjecture for parametrically kicked Hamilton systems, which says that for such systems the adiabatic invariant (the action) for an initial microcanonical ensemble at the mean final energy always increases under a parametric kick. I shall also present many examples of the validity of the PR property, which is almost always satisfied, but can be broken in not sufficiently smooth potentials or in cases where we are in the energy range close to a separatrix in the phase space. The general conjecture, using analytical and numerical computations, is shown to hold true for important systems like homogeneous power law potentials, pendulum, Kepler system, Morse potential, Pöschl-Teller I and II potentials, cosh potential, quadratic-linear potential, quadratic-quartic potential, while in three cases we demonstrate the absence of the PR property: Linear oscillator enclosed in a box, sextic potential, quartic double well potential. We shall discuss the physical relevance of these results and indicate some further research directions.

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# Dinamika hitrosti v časovno odvisnih biljardih

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Hitrost delca, ki se nahaja v časovno odvisnem biljardu, lahko neomejeno narašča. Takšnemu procesu pravimo Fermijevo pospeševanje, ki naj bi pojasnilo nastanek visoko energijskih kozmičnih delcev. Numerični računi kažejo, da povprečna hitrost  $v$ , ansambla delcev v časovno odvisnem biljardu, sledi potenčnemu zakonu  $v \propto n^\beta$ , kjer je  $n$  število trkov z robom biljarda. Eksponent pospeševanja  $\beta$  lahko ima vrednost med 0 in 1, ki je odvisna od dinamičnih lastnosti zamrznjenega biljarda in načina kako se biljard spreminja v času. Edine vrednosti  $\beta$ , ki jih teoretično razumemo so  $\beta = 1/2$  v generičnih kaotičnih biljardih [1],  $\beta = 1$  v primeru specialnih posebej konstruiranih biljardih [3,4] in vrednosti  $\beta = 1/4, 1/6$  in 0, ki jih dobimo v popolnoma kaotičnih časovno odvisnih biljardih, ki ohranjajo obliko in so odvisne le od ohranitve vrtilne količine biljarda [4]. Vrednosti  $\beta$  za primere, kjer je dinamika delca tudi regularna, še vedno teoretično ne razumemo. Primer  $\beta = 1$  ustreza eksponentno hitremu pospeševanju v odvisnosti od zveznega časa. V tej predstavitvi bom demonstriral kako lahko prepletanje med regularno in kaotično dinamiko privede do eksponentnega pospeševanja v do sedaj nepoznanem razredu gladkih časovno odvisnih biljardov.

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# Velocity dynamics in time-dependent billiards

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The velocity of a particle, confined inside a billiard with the moving boundary, may grow without limit. Such velocity gain is known as Fermi acceleration which was proposed as a mechanism for the creation of the high-energy cosmic particles. Numerical calculations show that the average velocity  $v$  of an ensemble of particles in a time-dependent billiard generally follows the power law  $v \propto n^\beta$ , with respect to the number of collisions  $n$ . The value of the acceleration exponent  $\beta$  is restricted to the interval between 0 and 1 and depends on the dynamical properties of the frozen billiard as well as on the driving law. The only values of  $\beta$  which are theoretically well understood are  $\beta = 1/2$  for generic fully chaotic billiards [1],  $\beta = 1$  for some specially designed billiards [3,4] and the values  $\beta = 1/4, 1/6$  and 0 which arise in the fully chaotic shape-preserving billiards and depend on whether the angular momentum of the billiard table is preserved or not [4]. The possible values of  $\beta$ , if the particle dynamics involves regular motion are still not understood. The  $\beta = 1$  case corresponds to the exponentially fast acceleration with respect to the continuous time. In this talk I will demonstrate how the interplay between the regular and chaotic motion may give rise to the exponential acceleration in a yet unknown class of smooth time-dependent billiards.

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# Nonlinear dynamics, wave chaos, and Bose-Einstein condensates

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Classical systems can be characterized by their integrability, nonintegrability, or chaoticity gradually showing rich and complex dynamics. A prominent example is the Fermi-Pasta-Ulam model of particles interacting via anharmonic forces modeling phonon coupling in a lattice. In the transition from a discrete system of individual particles to a continuous wave system some of these properties are preserved. An example for a continuous nonlinear wave equation is the Gross-Pitaevskii equation describing the dynamics of a Bose-Einstein condensate. I will show that this equation can be both integrable and chaotic depending on the external potential in which the condensate is propagating. I will discuss the physical interpretation of wave chaos within Bose-Einstein condensates.

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# 1 : -3 problem resonantnega centra za kvadratični sistem

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Iskanje potrebnih in zadostnih pogojev za obstoj lokalnega analitičnega prvega integrala diferencialnega sistema oblike

$$\dot{x} = x - a_{10}x^2 - a_{01}xy - a_{12}y^2, \quad \dot{y} = -3y + b_{21}x^2 + b_{10}xy + b_{01}y^2,$$

je tako imenovani 1 : -3 problem resonantnega centra za kvadratični sistem. Z delitvijo problema na dva podproblema, kjer je  $a_{01} = 0$  in  $a_{01} = 1$ , smo pridobili 25 primerov za  $a_{01} = 1$  in 11 primerov za  $a_{01} = 0$ . Potrebne pogoje smo pridobili s pomočjo modularne aritmetike. Glavna metoda, ki je bila uporabljena za dokaz zadostnosti pridobljenih pogojev, je bila Daurboux-ova metoda, vendar tudi druge tehnika so bile uporabljene.

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# The 1 : -3 resonant center problem in the quadratic case

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The 1 : -3 resonant center problem in the quadratic case is to find necessary and sufficient conditions for existence of local analytic first integrals of the differential system

$$\dot{x} = x - a_{10}x^2 - a_{01}xy - a_{12}y^2, \quad \dot{y} = -3y + b_{21}x^2 + b_{10}xy + b_{01}y^2.$$

Dividing this problem into two smaller problems, where  $a_{01} = 0$  and  $a_{01} = 1$ , we gain 25 center cases for  $a_{01} = 1$  and 11 cases for  $a_{01} = 0$ . The necessary conditions are obtained using modular arithmetics. The main tool to prove the sufficiency of the obtained conditions is the method of Darboux, however some other technics were used as well.

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# Collision scenaria of bodies in the early solar system

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It is well known that there is a daily bombardment of smaller bodies on the Earth which may cause even heavy damage (Chelyabinsk meteor); we have detailed studies of the collision which caused the Moon to form. We do not have such studies of early Solar system body collisions depending on their masses, their velocities and their compositions (rock, water, organic material, etc.). Hence we conduct an investigation to statistically determine the variety of impact scenaria for different bodies with respect to the former mentioned parameters. The possible formation of bigger objects and the transfer of water between the colliding bodies are then studied in detail with our SPH (Smooth Particle Hydrodynamics) code. We show results of this collision scenaria where the outcome depends on the masses involved, their encounter velocity, the impact angle, and their respective composition (basaltic rock, water).

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# Bifurkacije limitnih ciklov v ravninskih polinomskih sistemih

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Študij perturbacij integrabilnih sistemov je tesno povezan z enim izmed najbolj znanih problemov kvalitativne teorije dinamičnih sistemov - 16. Hilbertovim problemom o številu limitnih ciklov dvo-dimenzionalnih polinomskih sistemov  $\dot{x} = P_n(x, y)$ ,  $\dot{y} = Q_n(x, y)$  ( $n$  je maksimalna stopnja polinomov na desni strani sistema). Kljub dejstvu, da je ta problem bil formuliran več kot sto let nazaj, še ni rešen niti za kvadratične sisteme (sisteme z  $n = 2$ ). Bistven del problema je določitev maksimalnega števila limitnih ciklov, ki lahko bifurcirajo iz singularne točke tipa center ali fokus pod vplivom majhnih motenj koeficientov sistema, t.i. problem cikličnosti. Opisala bom redukcijo problema cikličnosti na algebraičen problem iskanja baze polinomskega ideala, generiranega s količinami, ki jih dobimo pri izračunu prvega integrala določene oblike. Ta ideal se imenuje Bautinov ideal in v primeru, ko je radikalni, je lahko problem cikličnosti rešen na relativno lahek način z uporabo algoritmov računske algebre. Problem pa je veliko težji v primeru neradikalnega Bautinovega ideala. Pristop, ki je lahko uspešno uporabljen za študij nekaterih sistemov z neradikalnim Bautinovim idealom je pred nedavnim bil predlagan v [2]. V našem študiju uporabimo ta pristop pri obravnavi cikličnosti družine kubičnih sistemov.

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# Bifurcations of limit cycles in plane polynomial systems

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The study of perturbations of integrable systems is closely related to one of the most famous problems in the qualitative theory of dynamical systems - Hilbert's sixteenth problem on the number of limit cycles of two dimensional polynomial systems  $\dot{x} = P_n(x, y)$ ,  $\dot{y} = Q_n(x, y)$  ( $n$  is the maximum degree of the polynomials on the right-hand side of the system). In spite of the fact that Hilbert's 16th problem was formulated more than a hundred years ago, it is not yet solved even for quadratic systems (systems with  $n = 2$ ). An essential part of the problem is the estimation of the maximum number of limit cycles which can bifurcate from a singular point of center or focus type under small perturbations of coefficients of the system, the so-called cyclicity problem. I will describe the reduction of the cyclicity problem to the algebraic problem of searching for a basis of a polynomial ideal generated by quantities computed by looking for formal first integral of required form. This ideal is called Bautin ideal and in the case that it is radical the problem of cyclicity can be solved in relatively easy way using algorithms of computational algebra. The problem is much more difficult in the case of non-radical Bautin ideal. An approach which can be successfully applied to studying some systems with non-radical Bautin ideal has been proposed recently in [2]. In our work we use it to study the cyclicity of a family of cubic systems.

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# Fizika supersimetrične kondenzirane snovi

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Študij močno sklopljenih fizikalnih sistemov pri neničelni temperaturi in gostoti predstavlja velik izziv v teoretski fiziki. Eno izmed redkih orodij, ki so nam na voljo za njihovo analizo, je umeritveno-gravitacijska (gauge-gravity) dualnost, ki slika vprašanja iz močno sklopljenih teorij polja na šibko sklopljene gravitacijske probleme. Dualnost je najlažje uporabiti v supersimetričnih teorijah in deformacijah le-teh, katerih pa še ne razumemo z vidika nizko-energetske teorije polja. Na predavanju bom predstavil supersimetrijo in nekatere nenavadne napovedi, ki nam jih je dala umeritveno-gravitacijska dualnost za supersimetrične sisteme kondenzirane snovi. Po tem bom obravnaval sistematični pristop k njihovemu opisu z vidika teorije polja in razloge, zakaj se ti sistemi ne obnašajo v skladu z našo intuicijo. Predstavil bom trditev, da je za to odgovorna predvsem prisotnosti skalarnih polj in njihove interakcije s fermioni. Osredotočil se bom na predstavitev stabilnosti supersimetričnega modulnega prostora pri neničelni gostoti in neobstoj Fermijevih ploskev v supersimetrični kvantni elektrodinamiki. Zaključil bom z obravnavo vprašanja, kaj nas lahko te teorije naučijo o stabilnosti D-bran v teoriji strun.

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# Supersymmetric condensed matter physics

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The study of strongly-coupled physical systems at finite temperature and density has been a major challenge in theoretical physics. One of the few available tools for their analysis is the gauge-gravity duality, which maps questions in strongly-coupled field theories to weakly-coupled gravity problems. However, the duality is most easily applied to supersymmetric field theories and their deformations, which have not been understood from the low-energy field theory point of view. In my talk, I will present supersymmetry and discuss some of the peculiar predictions gauge-gravity duality has made for supersymmetric condensed matter systems. I will then discuss a systematic approach to their field theoretic understanding and the reasons why these theories defy our conventional condensed matter intuition. I will argue that the main cause of its unintuitive behaviour is the presence of scalar fields and their interactions with fermions. I will focus on discussing the stability of the supersymmetric moduli space at finite density and the absence of Fermi surfaces in supersymmetric QED. I will conclude by discussing what these theories can teach us about the stability of D-brane configurations in string theory.

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# Rezultati eksperimenta ATLAS na LHC

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Od pomladi leta 2010 do konca leta 2012 je eksperiment ATLAS na Velikem hadronskem trkalniku zabeležil velik nabor podatkov ob trkih protonov pri težiščnih energijah do 8 TeV. Predstavil bom pregled zadnjih rezultatov meritev in prikazal odlično delovanje detektorja ATLAS. Predstavljeni bodo primeri analiz znotraj in onkraj Standardnega modela in priprave na nadaljnje raziskave. Odkritje Higgsovega bozona bo predstavljeno v drugem prispevku.

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# Results of the ATLAS Experiment at the LHC

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From spring 2010 till the end of 2012 the ATLAS experiment at the Large Hadron Collider has recorded a large statistics of proton-proton collision data at a centre-of-mass energy of up to 8 TeV. In this talk an overview of the latest results will be presented, demonstrating the outstanding performance of the ATLAS detector. Examples of Standard Model and Beyond SM analysis results will be given and the preparation for a wider range of searches will be discussed. The Higgs Boson discovery will be covered in a different contribution.

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# Two Examples of a Hard Exclusive Process

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In this talk we investigate two exclusive charm production processes within the generalized parton picture using a handbag-type mechanism: On the one hand the photoproduction of  $D$ -mesons, i.e. the reaction  $p \gamma \rightarrow \Lambda_c^+ \bar{D}^0$  and, on the other hand, the reaction  $p \pi^- \rightarrow \Lambda_c^+ D^-$ . We argue that under the physical plausible assumptions of restricted parton virtualities and intrinsic transverse momenta, the process amplitudes factorize into a perturbatively calculable partonic subprocess and hadronic matrix elements, which contain the non-perturbative bound-state dynamics of the hadronic constituents. These hadronic matrix elements are parameterized in terms of generalized parton distributions (GPDs) and a meson distribution amplitude (DA) for  $p \gamma \rightarrow \Lambda_c^+ \bar{D}^0$  and in terms of GPDs only for  $p \pi^- \rightarrow \Lambda_c^+ D^-$ . We can rely on previous work [1] where the  $p \rightarrow \Lambda_c^+$  transition GPDs were introduced. On the basis of a wave-function-overlap model for the generalized parton distributions, cf. Ref. [2], we obtain numerical predictions for  $p \rightarrow \Lambda_c^+$  transition form factors and for the differential and integrated cross section of the two processes we are interested in. The cross section measurements could determine the role of the intrinsic charm-quark content in the proton sea: The experimental finding of a much larger cross section as we get, could only be explained if the charm-quark content of the proton sea was non-negligible.

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# Novi detektorji za pozitronsko tomografijo

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V prispevku bom predstavil nove, zelo hitre detektorje žarkov gama, ki bi lahko znatno izboljšali slikanje s pozitronsko tomografijo, eno izmed najpomembnejših diagonostičnih metod. Z zelo hitrimi detektorji postane t.i. TOFPET slikanje, torej slikanje z meritvijo časa preleta fotonov, zelo hitra in natančna tehnika slikanja.

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# Novel detectors for positron emission tomography (PET)

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This work investigates the possibilities of improving the measurements of arrival time difference of the two 511 keV photons arising from annihilation of a positron in positron emission tomography (PET). The new technique of detecting the prompt Cherenkov light, produced by absorption of the annihilation photon in a suitable crystal, could considerably improve the image quality. A simple apparatus with PbF<sub>2</sub> crystals and microchannel plate photomultipliers (MCP PMTs) has been constructed and coincidence resolutions of 71 ps FWHM and 95 ps FWHM have been achieved with 5 and 15 mm thick crystals, respectively. Simulation calculations are in agreement with the experimental findings.

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# Probing the role of accelerator modes on the dynamical localization properties of the quantum kicked rotator and on the anomalous diffusion of its classical analogue.

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In the first part of the talk, we study the  $N$ -dimensional model of the quantum kicked rotator in the classically fully chaotic regime, which in the limit of sufficiently large  $N$  tends to the quantized kicked rotator. We describe the features of dynamical localization of chaotic eigenstates as a paradigm for other both time-periodic and time-independent (autonomous) fully chaotic or/and mixed type Hamilton systems. We generalize the scaling variable to the case of anomalous diffusion in the classical phase space, by deriving the localization length for the case of generalized classical diffusion. We greatly improve the accuracy and statistical significance of the numerical calculations, giving rise to the following conclusions: (i) The level spacing distribution of the eigenphases is very well described by the Brody distribution, systematically better than by other proposed models. (ii) We study the eigenfunctions of the Floquet operator and characterize their localization properties using the information entropy measure. (iii) We show the existence of a scaling law between the localization parameter and the relative localization length, now including the regimes of anomalous diffusion.

In the second part, we focus on the effect of the anomalous diffusion arising due to the accelerator modes in the classical kicked rotator, exemplified by the standard map. The systematic approach rests upon detecting the regular and chaotic regions

and thus to describe in detail the structure of the phase space, the description of the momentum distribution in terms of the Lévy stable distributions, the numerical calculation of the diffusion exponent and of the corresponding diffusion constant, and the various kinds of correlation functions. We use this approach to analyze in detail and systematically the standard map at all values of the kick parameter  $K$ , up to  $K = 70$ . All complex features of the anomalous diffusion are well understood in terms of the role of the accelerator modes, mainly of period 1 at large  $K \geq 2\pi$ , but also of higher periods (2,3,4,...) at smaller values of  $K \leq 2\pi$ .

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# S pristopi teorije grafov do razumevanja funkcionalne povezanosti med celicami beta v Langerhansovih otočkih

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Od odkritja topoloških lastnosti mrež malega sveta in skalno neodvisnih mrež v številnih realnih sistemih smo priča razmahu raziskav s področja kompleksnih mrež. To relativno novo raziskovalno področje se vse pogosteje uporablja za preučevanje in razumevanje organizacijskih in funkcionalnih principov različnih kompleksnih sistemov. V tem prispevku se osredotočimo na uporabo pristopov teorije kompleksnih mrež za raziskovanje narave funkcionalne povezanosti v živih tkivih, v našem primeru trebušne slinavke miši, v kateri se nahajajo Langerhansovi otočki, ki so majhni organi iz do nekaj tisoč med seboj povezanih celic beta, ki izločajo inzulin in tako igrajo osrednjo vlogo v telesni homeostazi energijsko bogatih molekul. Funkcionalno povezanost smo določili na podlagi korelacij med časovnimi vrstami znotrajceličnih kalcijevih signalov v celicah beta trebušne slinavke miši, posnetih s konfokalno mikroskopijo. Analiza ekstrahiranih funkcionalnih mrež je pokazala, da so topološke lastnosti mreže in doseg interakcij močno odvisne od stimulatorne koncentracije glukoze. Eksperimenti, pri katerih se stimulatorna koncentracija korakoma zvišuje, nam omogočajo preučevanje dinamične evolucije mrež. Naši rezultati so pokazali na obstoj jasno izraženih lokalnih skupnosti v strukturi funkcionalne povezanosti, kar kaže na razdrobljeno organiziranost sincicija celic beta v Langerhansovem otočku trebušne slinavke. Uporabljeni teoretični pristopi omogočajo detekcijo najbolj pomembnih celic v smislu iniciatorjev in mediatorjev signalov. Izkazalo se je, da se te pomembne vloge nenehno menjujejo med deli tkiva, kar kaže na obstoj mehanizmov "delitve nalog". Predstavljeni rezultati vodijo do novih in poglobljenih spoznanj s področij mehanizma delovanja in funkcionalne organiziranosti sincicija celic beta, ki s konvencionalnimi metodološkimi orodij ne bi bila mogoča.

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# Graph-theoretical approaches reveal the nature of functional connectivity patterns between beta cells in Islets of Langerhans

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Complex networks have received a great amount of interest since the discovery of scale-free and small-world topological features in a plethora of real world systems. Approaches developed in the field of network science are nowadays widely used for the analysis of various complex systems. In the present study, we utilize these approaches for investigating the functional connectivity patterns in living tissues, such as mouse pancreatic tissue in which islets of Langerhans are embedded. These microorgans consist of up to a few thousand interconnected beta cells that secrete insulin and play a pivotal role in whole body energy homeostasis. The networks were extracted on the basis of correlation between intracellular calcium signals from a population of beta cells, obtained by confocal microscopy of fluorescently labeled beta cells in acute mouse pancreas tissue slices. By analyzing resulting functional networks, we showed that topological features as well as the range of interactions depend strongly on the stimulatory glucose concentration. On the basis of experimental measurements in which the glucose concentration is increased step-wise, we were able to trace the temporal evolution of the underlying network. Our results revealed that well-pronounced local communities characterize the functional network topology, thereby indicating a dispersed organization of the syncytium. Furthermore, our theoretical approaches enabled us to detect the most important initiator and mediator cells in the tissue. It turned out that these important roles are continuously switching among parts of the tissue, which implies the existence of a task-sharing-like mechanism in the islets of Langerhans. Presented results shed novel insight into the functional mechanisms and organization of beta cell syncytia, which could not be obtained with conventional methodological tools.

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# Odkritje Higgsovega bozona z detektorjem ATLAS na Velikem hadronskem trkalniku

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V juliju 2012 sta poskusa ATLAS in CMS na Velikem hadronskem trkalniku LHC v CERNu skupno najavila odkritje težkega bozona z maso okoli 126 GeV. Nadaljnja analiza trkov protonov na LHC iz let 2011 in 2012 je razkrila lastnosti delca, ki so v celoti skladne s tistimi, ki so napovedane za Higgsov bozon, zadnji manjkajoči člen Standardnega modela. Letošnja Nobelova nagrada za fiziko je bil podeljena F. Englertu in P. Higgsu za njun teoretični prispevek k izvoru mase, ki je daljnega leta 1964 napovedoval, da v Naravi obstaja skalarni delec, kasneje poimenovan Higgsov bozon. V predavanju bom predstavil razloge za skoraj 50-letni časovni zamik med Higgsovo napovedjo in lanskoletnim odkritjem. Podrobno bom predstavil detektor s poudarkom na delih, bistvenih za odkritje Higgsovega bozona, v treh glavnih razpadnih kanalih:  $H \rightarrow \gamma\gamma$ ,  $H \rightarrow ZZ^* \rightarrow 4l$  in  $H \rightarrow WW^* \rightarrow l\nu l\nu$ . V nadaljevanju bom izpostavil vpliv odkritja na naše razumevanje Narave in eksperimentalne izzive, ki jih predstavlja na novo odkrit delec za prihodnost.

# Discovery of the Higgs Boson with the ATLAS Detector at the Large Hadron Collider

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In July 2012 ATLAS and CMS experiments at the Large Hadron Collider issued a joint announcement of the discovery of a heavy boson with a mass around 126 GeV. Further analysis of LHC  $pp$  collision data from 2011 and 2012 revealed its properties, which appear fully consistent with those predicted for the Higgs boson, the last missing constituent of the Standard Model. This years Nobel Prize in Physics has been awarded to F. Englert and P. Higgs for their theoretical contribution to the origin of mass back in 1964 that postulated the existence of a scalar particle, later named the Higgs boson. In the lecture I will outline reasons for the close to 50-year time lag between the Higgs prediction and last years discovery. I will detail the experimental apparatus focusing on the parts essential for the Higgs discovery in the three main decay channels:  $H \rightarrow \gamma\gamma$ ,  $H \rightarrow ZZ^* \rightarrow 4\ell$  and  $H \rightarrow WW^* \rightarrow \ell\nu\ell\nu$ . I will expose the impact of the discovery on our understanding of Nature and elaborate on the experimental challenges the newly discovered particle sets for the future.

# Flavor contents in the nucleons and hyperons

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Recently the compositions of the nucleons in terms of their quark flavors have been analyzed phenomenologically from the world data base of elastic electron scattering on the proton and the neutron [1]. The corresponding results provide rather stringent tests for any theory of low-energy nucleons: Their electromagnetic form factors must be described in accordance with the existing global data and also their flavor decompositions. We have studied the electromagnetic form factors of the nucleons quite some time ago along the relativistic constituent-quark model based on Goldstone-boson-exchange dynamics [2], finding surprisingly good agreement of its covariant predictions for the nucleon electroweak structures with existing experimental data [3]. Now we have subject the model also to testing its flavor content. Again, we obtain a reasonable description of the various flavor contributions to the electricomagnetic form factors in practically all instances [4]. It means that at low and moderate momentum transfers the nucleons can well be described as three-quark states with flavors  $u$  and  $d$  only. The same analysis has also been extended to other baryons in the realm of  $u$ ,  $d$ , and  $s$  flavors, i.e. the  $\Delta$  and the hyperons [5,6]. In several respects quite surprising insights are obtained their flavor contents.

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# Neravnovesne lastnosti izolatorjev Motta in Hubbarda

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V predavanju bom predstavil problematiko in nekatere naše rezultate teorije izolatorjev Motta in Hubbarda, zlasti glede njihovih anomalnih lastnosti in obnašanja v neravnovesju. Model enodimenzionalne verige fermionov pokaže, da je odziv na zunanje polje v prevodnem kot tudi v izolatorskem režimu anomalen v integrabilnem primeru, za razliko od normalnega odziva v generičnem neintegrabilnem sistemu. Študija dielektrična preboja v polariziranem Hubbardovem modelu omogoči točen izračun prebojnega polja in njegove odvisnosti od energijske vrzeli. Predstavljena bo tudi analiza rekombinacije fotoinduciranih nabojev v izolatorjih Motta in Hubbarda. Teorija pokaže, da je možna interpretacija eksperimentov ultrahitre spektroskopije na nedopiranih kupratih v povezavi s procesi več-magnonske emisije.

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# Nonequilibrium properties of Mott-Hubbard insulators

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In the talk I will discuss some aspects of the anomalous behaviour of Mott-Hubbard insulators in a nonequilibrium situations. A model of one-dimensional interacting fermions in the conducting as well in the insulating regime shows that response is anomalous for the integrable case as opposed to generic nonintegrable models. The study of the dielectric breakdown within the polarized Hubbard model gives an exact dependence of the threshold field on the excitation gap. Finally, the recombination of photoinduced charges in Mott-Hubbard insulators will be evaluated and theory involving the multi-magnon emission applied to the interpretation of ultrafast spectroscopy experiments on undoped cuprates.

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# Difuzijski transport v integrabilnih sistemih

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Rigorozna utemeljitev makroskopskih difuzijskih transportnih zakonov na podlagi mikroskopskih enačb gibanja je eno glavnih odprtih vprašanj neravnovesne statistične fizike. Problem je še posebej pereč v enodimenzionalnih sistemih z lokalno interakcijo, kjer numerični poskusi kažejo na zelo pestro obnašanje transporta toplote, delcev, ali magnetizacije.

V predavanju bom najprej pokazal nekaj računskih eksperimentov, ki kažejo na zelo zanimivo (in do nedavna nepredstavljivo) možnost difuzijskega transporta v popolnoma integrabilnih sistemih, tako v klasični [1] kot kvantni [2,3] mehaniki. V kvantni statistični mehaniki nam robno gnana Lindbladova enačba ponuja nove možnosti za iskanje točnih rešitev stacionarnih stanj daleč od ravnovesja. Posebej obetavna se zdi nedavna točna rešitev odpre Hubbardove verige [4].

V zadnjem delu predavanja bom pokazal splošno kinematično neenakost [5] (glej tudi [6]), ki nam omogoča strogo navzdol oceniti visokotemperaturne difuzijske konstante s pomočjo kvadratično ekstenzivnih konstant gibanja, ki obstajajo v nekaterih integrabilnih sistemih, npr. v Heisenbergoviali Hubbardovi verigi.

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# Diffusive transport in integrable systems

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Rigorously establishing macroscopic diffusive transport laws on the basis of microscopic equations of motion is one of the main unsettled issues of non-equilibrium statistical physics. The problem is particularly severe in one-dimensional locally interacting systems where numerical experiments show a rich variety of behaviors of the transport of heat, particles, and magnetization.

In the talk I will first demonstrate several computer simulations which indicate a very interesting (and until recently, unseen) possibility of diffusive transport in completely integrable systems, both, in classical [1] and quantum [2,3] mechanics. In quantum statistical mechanics, the boundary driven Lindblad equation offers new possibilities for constructing exact solutions of stationary states far from equilibrium. Particularly interesting seems a recent solution of the open Hubbard chain [4].

In the last part of the talk I will show a general kinematic inequality [5] (see also [6]), which yields strict lower bounds on high-temperature diffusion constants in terms of quadratically extensive constants of motion, which exist in certain integrable systems, e.g., in the Heisenberg spin 1/2 chain or in the Hubbard chain.

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# Točne ne-adiabatne ne-Abelove geometrijske faze

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Najprej bomo predstavili točno rešitev za valovno funkcijo elektrona v polprevodniški kvantni žici s spinsko-tirno sklopitvijo. Elektron poganjamo s časovno odvisnim harmonskim potencialom [1]. Motivacija je manipulacija spina elektrona z lokalnim zunanjim električnim poljem – brez dodatnih magnetnih polj, ki jih ni možno uporabiti na majhnem področju.

Nato bomo rešitev razširili na bolj splošen sistem, kjer je časovno odvisna tudi spinsko-tirna sklopitvena konstanta. Ta dodatna, od časa odvisna, prostostna stopnja omogoča holonomno ne-Abelovo manipulacijo spina elektrona. Za širok nabor časovno odvisnih gonilnih funkcij lahko s pomočjo točne rešitve tudi v ne-adiabatnem področju analitično izrazimo dinamično in geometrijsko Anandanovo fazo [2], ki se v adiabatski limiti zreducira na fazo Wilczka in Zeeja [3]. Z zlomitvijo simetrije časa predstavljajo rezultati rešitev za fazo Aharonova in Anandana [4], ki nato v adiabatski limiti preide v običajno Berryjevo fazo [5]. Podali bomo tudi kratek uvod v pojem geometrijske faze v kvantni mehaniki.

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# Exact non-adiabatic non-Abelian geometric phases

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First we will present an exact solution for the wavefunction of an electron in a semiconductor quantum wire with spin-orbit interaction and driven by external time-dependent harmonic confining potential [1]. The motivation is the manipulation of electron spin by locally applying an external electric field – in the absence of magnetic fields which in practice can not selectively be applied in spatially small regions.

Next, the solution will further be extended to a more general system, where also the spin-orbit interaction can be time dependent. This additional time dependent degree of freedom enables a holonomic non-Abelian qubit manipulation. For a broad class of driving functions one can by the virtue of the exact solution also in the non-adiabatic regime construct analytically the corresponding dynamical and the geometric Anandan phase [2] or in the adiabatic limit the Wilczek-Zee phase [3]. By breaking the time reversal symmetry the results lead to the corresponding Aharonov-Anandan phase [4] which in the adiabatic limit reduces to the usual Berry phase [5]. A short introduction to the concept of geometric phases in quantum mechanics will be given.

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# Kvantna lokalizacija kaotičnih stanj in statistika energijskih spektrov

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Kvantna lokalizacija klasično kaotičnih lastnih stanj je eden najpomembnejših pojavov v kvantnem kaosu, ali bolj splošno - v valovnem kaosu, poleg značilnega vedenja statističnih lastnosti energijskih spektrov. Kvantna lokalizacija nastopi, če je t.i. Heisenbergov čas  $t_H$  danega sistema krajši od vseh klasičnih transportnih časov v danem klasičnem sistemu, se pravi, kadar je klasični transport počasnejši od kvantne časovne resolucije evolucijskega operatorja. Heisenbergov čas  $t_H$ , kot pomembna karakteristika vsakega kvantnega sistema, je namreč enak razmerju Planckove konstante  $2\pi\hbar$  ter srednjega razmika med energijskimi nivoji  $\Delta E$ ,  $t_H = 2\pi\hbar/\Delta E$ .

Pokazali bomo funkcionalno povezavo med stopnjo lokalizacije ter spektralno statistiko v avtonomnih (časovno neodvisnih) sistemih, v analogiji z brcanim rotorjem, ki pa je paradigma časovno periodičnih (Floquetovih) sistemov [7], ter pristop in metodo ilustrirali v primeru družine biljardov [8,9] v dinamičnem režimu med integrabilnostjo (krog) in polnem kaosom (kardioda), kjer bomo ekstrahirali kaotična stanja. Stopnjo lokalizacije določimo z dvema lokalizacijskima merama, z uporabo t.i. Poincaré-Husimijevih funkcij (ki so Gaussovo glajene Wignerjeve funkcije v Poincaré-Birkhoffovem faznem prostoru), ki so pozitivno definitne in jih lahko obravnavamo kot kvazi-verjetnostne gostote. Prva mera  $A$  je definirana s pomočjo informacijske entropije, druga ( $C$ ), pa s pomočjo korelacij v faznem prostoru Poincaré-Husimijevih funkcij lastnih stanj. Presenetljivo in zelo zadovoljivo se izkaže, da sta obe meri linearno povezani in zato ekvivalentni.

Ena od glavnih manifestacij kaosa v kaotičnih stanjih v odsotnosti kvantne lokalizacije je porazdelitev  $P(S)$  razmikov  $S$  med sosednjimi energijskimi nivoji, ki se vede pri majhnih  $S$  kot linearna funkcija  $P(S) \propto S$ , in govorimo o linearnem odbijanju med sosednjimi nivoji, medtem ko imamo v integrabilnih sistemih Poissonovo statistiko (eksponentno funkcijo  $P(S) = \exp(-S)$ ), kjer odbijanja med sosednjimi nivoji ni ( $P(0) = 1 \neq 0$ ). V povsem kaotičnem režimu s kvantno lokalizacijo pa opazimo, da je  $P(S)$  pri majhnih  $S$  potenčna funkcija  $P(S) \propto S^\beta$ , z  $0 < \beta < 1$ . Pokazali

bomo, da obstaja funkcionalna odvisnost med mero lokalizacije  $A$  in eksponentom  $\beta$ , namreč da je  $\beta$  monotona funkcija  $A$ : pri močni lokalizaciji sta  $A$  in  $\beta$  majhna, pri šibki lokalizaciji (skoraj povsem razširjena kaotična stanja) pa sta  $A$  in  $\beta$  blizu 1.

Pristop ilustriramo v primeru kaotičnih stanj za zgoraj omenjene družine biljardov, kjer lahko ločimo regularna in kaotična stanja. Ta predstavitev sloni na naših najnovejših člankih [1,4,5,7].

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# Quantum localization of chaotic eigenstates and the statistics of energy spectra

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Quantum localization of classical chaotic eigenstates is one of the most important phenomena in quantum chaos, or more generally - wave chaos, along with the characteristic behaviour of statistical properties of the energy spectra. Quantum localization sets in, if the Heisenberg time  $t_H$  of the given system is shorter than the classical transport times of the underlying classical system, i.e. when the classical transport is slower than the quantum time resolution of the evolution operator. The Heisenberg time  $t_H$ , as an important characterization of every quantum system, is namely equal to the ratio of the Planck constant  $2\pi\hbar$  and the mean spacing between two nearest energy levels  $\Delta E$ ,  $t_H = 2\pi\hbar/\Delta E$ .

We shall show the functional dependence between the degree of localization and the spectral statistics in autonomous (time independent) systems, in analogy with the kicked rotator, which is the paradigm of the time periodic (Floquet) systems [7], and shall demonstrate the approach and the method in the case of a billiard family [8,9] in the dynamical regime between the integrability (circle) and full chaos (cardioid), where we shall extract the chaotic eigenstates. The degree of localization is determined by two localization measures, using the Poincaré Husimi functions (which are the Gaussian smoothed Wigner functions in the Poincaré Birkhoff phase space), which are positive definite and can be treated as quasi-probability densities. The first measure  $A$  is defined by means of the information entropy, whilst the second one,  $C$ , in terms of the correlations in the phase space of the Poincaré Husimi functions of the eigenstates. Surprisingly, and very satisfactory, the two measures are linearly related and thus equivalent.

One of the main manifestations of chaos in chaotic eigenstates in absence of the quantum localization is the energy level spacing distribution  $P(S)$  (of nearest neighbours), which at small  $S$  is linear  $P(S) \propto S$ , and we speak of the linear level repulsion, while in the integrable systems we have the Poisson statistics (exponential function  $P(S) = \exp(-S)$ ), where there is no level repulsion ( $P(0) = 1 \neq 0$ ). In

fully chaotic regime with quantum localization we observe that  $P(S)$  at small  $S$  is a power law  $P(S) \propto S^\beta$ , with  $0 < \beta < 1$ . We shall show that there is a functional dependence between the localization measure  $A$  and the exponent  $\beta$ , namely that  $\beta$  is a monotonic function of  $A$ : in the case of the strong localization are  $A$  and  $\beta$  small, while in the case of weak localization (almost extended chaotic states)  $A$  and  $\beta$  are close to 1.

We shall illustrate the approach in the model example of the above mentioned billiard family, where we can separate the regular and chaotic states. This presentation is based on our very recent papers [1,4,5,7].

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# Integrabilnost in bifurkacije limitnih ciklov v polinomskih sistemih NDE

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Obravnavamo polinomsko vektorsko polje na  $\mathbb{R}^n$  z neizrojeno singularno točko v izhodišču koordinatnega sistema. Razpravljaj bom o problemih lokalne analitične integrabilnosti, centralne mnogoterosti, problemu stabilnosti in medsebojno povezavo teh problemov. Predstavil bom izračun centralne mnogoterosti in prvih integralov nekaterih družin tri-dimenzionalnih in štiri-dimenzionalnih sistemov s kvadratičnimi in kubičnimi nelinearnosti. Povedal bom o računsko učinkovitih pristopih za študij takšnih problemov. Nekaj pozornosti bom namenil bifurkacijam limitnih ciklov v zgoraj omenjenih sistemih.

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# Integrability and bifurcations of limit cycles in polynomial systems of ODEs

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Consider polynomial vector fields on  $\mathbb{R}^n$  having a singularity at the origin with non-degenerate linear part. We discuss the problems of existence of analytic first integrals, center manifolds, periodic solutions and the problem of stability, and their interconnection. We also explicitly compute center manifolds and first integrals for several families of 3-dim and 4-dim systems with quadratic and cubic higher order terms and describe computational methods for the study. Bifurcations of limit cycles are discussed as well.

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# Quantum Difference–Differential Equations

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Differential equations which contain the parameter of a scaling process are usually referred to by the name Quantum Difference–Differential Equations. Some of their applications to discrete models of the Schrödinger equation are presented and some of their rich, filigrane und sometimes unexpected analytic structures are revealed.

A Lie-algebraic concept for obtaining basic adaptive discretizations is explored, generalizing the concept of deformed Heisenberg algebras by Julius Wess. They are also related to algebraic foundations of quantum groups in the spirit of Ludwig Pittner.

Some of the moment problems of the underlying basic difference equations are investigated. Applications to discrete Schrödinger theory are worked out and some spectral properties of the arising operators are presented, also in the case of Schrödinger operators with basic shift–potentials and in the case of ground state difference–differential operators.

For the arising orthogonal function systems, the concept of inherited orthogonality is explained. The results in this talk are mainly related to a recent joint work with Sophia Roßkopf and Lucia Birk.

Following a suggestion by Hans–Jürgen Stöckmann from the last Christmas symposium in Maribor 2012, the analogous situation on an equidistant lattice has now been worked out and leads to some amazing effects. These new results will also be presented.

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# Mesoscopic Physics with Ultracold Atoms: From Correlated Tunneling to Confinement-Induced Transparency

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Ultracold atomic and molecular physics has over the past decade branched out into a number of independent subfields. We provide here an overview of mesoscopic aspects of ultracold atomic structures and processes in tightly confining traps. For strongly interacting ultracold atoms tunneling represents a major mechanism for the transport and nonequilibrium dynamics of the atoms as well as the formation of novel quantum phases. At hand of several examples we demonstrate novel fundamental tunneling processes, some of them being counterintuitive, such as the tunneling of repulsively bound atomic clusters. Conditional, counterflow and excited band tunneling belong to the toolbox of modern ultracold physics and represent the first steps towards an atomtronic implementation of microscopic processes [1-4]. Opposite to ground state atoms Rydberg atoms show long-range interactions and possess an extreme polarizability which makes them highly susceptible to external fields. We show here that Rydberg physics in optical lattices opens the pathway for an intriguing excitation dynamics allowing to dynamically probe e.g. crystalline phases of Rydberg lattices [5-7]. Finally we address the possibility of introducing confinement-induced effects meaning effects emerging in tightly confining waveguides on the ultracold scattering of bosons or fermions. Here the principal mechanisms for induced resonances and transparency as well as molecule formation are discussed as exemplary processes which enrich the physics of the 'many-body landscape' [8-10].

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# Sončno obsevanje in klimatske spremembe po Milankovičevem modelu

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Predstavljen bo preprost model, ki reproducira osnovno obliko klimatske krivulje za obdobje würmske poledenitve. Osnova klimatskih modelov je namreč toplota, ki jo Zemlja prejme od Sonca. V prispevku bodo nazorno izpeljane enačbe, ki vodijo do dnevnega obseva. Z analitičnimi približki v najnižjem redu bo upoštevano spreminjanje nagiba zemeljske vrtilne osi in ekscentričnosti Zemljinega tira.

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# Solar insolation and climatic variations according to Milankovitch model

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A simple model will be introduced that reproduces the main shape of the climatic curve for the period of Würm ice age. Climatic models are essentially based on the energy received by Earth from the Sun. The contribution will show transparent derivation of equations that lead to the daily insolation. Analytical approximations are proposed for the time variation of the tilt of the geographical axis with respect to the ecliptic and of eccentricity of the Earth's orbit.

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# Kronotaksični sistemi: kaj so in zakaj so?

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Do nedavnega so bili deterministični neavtonomni oscillatorni sistemi s stabilnimi amplitudami in časovno-spremenljivimi frekvencami pogosto obravnavani kot stohastični sistemi. Mi smo take sisteme poimenovali kronotaksične. V nasprotju z konvencionalnimi modeli samodejnih oscilatorjev z limitnimi cikli, ti sistemi imajo časovno-odvisni točkast atraktor, oziroma stacionarno stanje. To omogoča oscillatornim gibanjem s časovno-spremenljivimi frekvencami da se uprejo motnjam, pojav ki je navzoč v vseh živih sistemih. Podrobno bomo predstavili teoretično osnovo kronotaksičnih sistemov, zlasti v primeru ločene amplitudne in fazne dinamike, in nakazali kako lahko izluščimo njihove značilnosti iz izmerjenih časovnih vrst.

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# Chronotaxic systems: What and why?

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Until recently, deterministic non-autonomous oscillatory systems with stable amplitudes and time-varying frequencies were not recognised as such and have often been mistreated as stochastic. A new class of systems, named chronotaxic, was recently introduced. In contrast to conventional limit cycle models of self-sustained oscillators, these systems possess a time-dependent point attractor or steady state. This allows oscillations with time-varying frequencies to resist perturbations, a phenomenon which is ubiquitous in living systems. A detailed theory of chronotaxic systems will be presented, and inverse approach to these systems will be formulated. It will be shown how observed data arranged in a single-variable time series can be used to determine whether or not a system is chronotaxic.

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# Degenerate parametric amplification of multiparameter squeezed photons

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In the Schrödinger picture, we discuss explicit solutions for two models of degenerate parametric oscillators in the case of multiparameter squeezed input photons. The corresponding photon statistics and Wigner's function are also derived in coordinate representation. Their time evolution is investigated in detail.

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# Modulirane faze v kiralnih nematičnih tekočih kristalih

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Že leta 1973 je R. Meyer izpostavil [1], da fleksoelektrični pojav v nematičnih tekočih kristalih lahko stabilizira prostorsko modulirane faze. Ker pa je za stabilizacijo moduliranih faz potreben zelo velik fleksoelektričnih koeficient, kar nekaj redov velikosti višji kot pri tekočih kristalih, ki so jih takrat poznali, je znanstvena srednja na modulirane nematične faze praktično pozabila. Idejo je oživil Dozov leta 2001 [2], ko je napovedal, da je elastična konstanta za upogibno deformacijo lahko negativna, če imamo tekoči kristal, ki ga tvorijo molekule z ukrivljeno sredico. Ker so meritve pokazale, da je fleksoelektrični koeficient pri tovrstnih tekočih kristalih zares izjemno velik [3], je fleksoelektrični pojav primeren kandidat za renormalizacijo upogibne elastične konstante [4], ki pod neko kritično temperaturo lahko postane negativna.

Eksperimentalno so prostorsko modulirane nematične faze najprej opazili Panov in sod. [5], temu pa je sledila množica raziskav v želji pojasniti obnašanje teh fascinantnih faz. V letu 2013 so kar tri raziskovalne skupine neodvisno poročale o moduliranih nematičnih fazah z izjemno kratko modulacijsko periodo, ki znaša le nekaj dolžin molekul [6-8].

V splošnem lahko obstajata dve vrsti moduliranih faz: zvojno-upogibna in pahljačasto-upogibna. V akiralnih nematikih je stabilna le ena, odvisno od razmerja med elastično konstanto za pahljačasto in zvojno deformacijo. Ker je elastična konstanta za pahljačasto deformacijo v nematikih iz ukrivljenih moleku zelo visoka, so do sedaj opazili zgolj fazo z zvojno-upogibno modulacijo.

Kiralni nematiki, ki jih tvorijo kiralne ukrivljene molekule ali kirani dimeri, pa imajo

izjemno kompleksno fazno obnašanje. Zep in sod. [9] poročajo o šestih nematičnih fazah, ki so jih opazili v temperaturnem območju med izotropno in kristalno fazo. Štiri od opaženih faz obstajajo v temperaturnem območju, ki je ožje od 1 K, kar kaže na to, da v sistemu tekmujejo številne interakcije. Visokotemperaturna nematična faza je klasična holesterična faza, medtem ko ima nizkotemperaturna nematična faza pahljačasto-zvojno modulirano strukturo, ki jo identificiramo tako, da opazujemo fokusiranje svetlobe zaradi periodičnega spreminjanja lomnega količnika materiala. Dolžina modulacije je v območju mikrometrov in se veča, če večamo debelino celice, v kateri je tekoči kristal.

Na predavanju bom predstavila teoretični model za opis strukture moduliranih faz v kiralnih nematikih. Model razloži zgoraj navedene lastnosti in predpostavi obstoj obeh moduliranih faz, zvojno-upogibna in pahljačasto-upogibna, v isti snovi in fazni prehod med njima. Model tudi napove, da z racemizacijo vzorca lahko pričakujemo prehod kiralne modulirane strukture z modulacijo reda mikrometrov v akiralno modulirano strukturo z modulacijo reda velikosti nanometrov. Opažanje takega faznega prehoda še predstavlja izziv za eksperimentalce.

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# Splay-bend and twist-bend modulated phases in chiral nematic liquid crystals

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The flexoelectric effect can be a driving mechanism for the formation of the twist-bend and splay-bend modulated liquid crystal nematic phases, as pointed out already in 1973 [1]. Because very large flexoelectric coefficients are required for the modulated structure to be stabilized, the idea of modulated nematic phases was practically forgotten until 2001 when Dozov [2] revived it by noting that, due to the specific shape of the constituent molecules, liquid crystals made of bent-core molecules can have negative bend elastic constants, which would promote modulated structures. Because in nematic liquid crystalline phases made of bent-core or dimer molecules, the flexoelectric coefficient can be expected to be very large [3] the flexoelectric effect is a proper candidate responsible for the renormalization of the bend elastic constant [4], which can become negative below some critical temperature.

The first experimental observation of the nematic to modulated nematic phase transition was reported by Panov *et al* [5], followed by an extensive research of several groups in the quest of understanding these fascinating phases. Just recently the structure of the modulated phase has been described in detail by three research groups, which independently found modulated nematics with an extremely short modulation period of only few molecular lengths [6-8].

There are, in general, two modulated nematic phases: twist-bend and splay-bend phase. In achiral nematics one can observe only one of the modulated structures, depending on the ratio between the splay and twist elastic constant. Because the splay elastic constant in bent-core liquid crystal is very large, only the twist-bend structures were observed so far.

On the other hand, chiral nematics made of chiral bent-core molecules or chiral dimers show an extremely complex phase behavior. As reported by Zep *et al* [9], six nematic type phases, all without the long range positional order, were observed in chiral dimers in a temperature range between the isotropic and crystal phase. Four of the observed phases exist in a temperature range of less than 1 K, which suggests that there are several strongly competing interactions in the system. The highest temperature phase is a regular uniaxial cholesteric phase, while the lowest temperature nematic phase has the splay-bend modulated structure that is easily identified by its stripe texture due to the focusing of light on the periodic changes of refractive index in the material. The pitch of this modulation is in a micrometer range and it increases with increasing cell thickness, which is opposite to the effect observed in ferroelectric smectic C phase, where the modulation periodicity increases with decreasing cell thickness as a result of helix unwinding caused by surface interactions.

In the talk we will report on the theoretical model of modulated structures in chiral nematics. The model explains the above-mentioned observed properties of nematic phases formed by chiral dimer molecules, it predicts the existence of the twist-bend and splay-bend structure in the same material and a phase transition between them and, finally, it predicts that racemization of the chiral nematics could lead to a decrease of the modulation pitch from the micrometer to the nanometer scale, an experimental quest left open to be observed.

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# Metode za optimizacijo tridimenzionalnih aerodinamičnih površin

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Predstavil bom več metod optimizacije aerodinamičnih površin, razvitih in uporabljenih na podjetju Pipistrel. S primernim zapisom optimizacijskega kriterija in parametrizacijo površine je mogoče zasnovati optimalne oblike kril, propelerjev, krilnih profilov ter trupov, pa tudi detajlov kot so prehod med krilom in trupom. Predvsem se bom osredotočil na dve metodi. S prvo metodo je mogoče najti optimalno obliko krila, ki ne leži v ravnini (npr. krilni zavihki) in kjer se izkaže, da ima prehod optimalnosti ravninske proti neravninski obliki kritično točko glede na parameter, ki določa razmerje med profilnim in induciranim uporom. Z drugo metodo je z uporabo modela idealne tekočine ter optimizacijskega kriterija, ki počiva na porazdelitvi tlaka po površini, mogoče najti tridimenzionalne oblike, ki stremijo k čim bolj laminarnemu obtekanju ter preprečujejo odcep toka, kar oboje znižuje upor.

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# Threedimensional aerodynamic surface optimisation methods

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I will present multiple methods for aerodynamic surface optimisation that were developed and are being used at Pipistrel. By a proper formulation of the optimisation criterion and parametrisation of surfaces, it is possible to design optimal wings, propellers, airfoils and fuselages, but also details such as wing root blending. I will mostly focus on two methods. The first method is used to find the optimal non-planar shape of the wing (e.g. winglets), where it is demonstrated that there exists a critical point determining the optimality of the planar versus nonplanar configuration, depending on the parameter that determines the ratio between profile and induced drag. The second method uses an ideal fluid model and a pressure distribution based optimisation criterion. It is used to obtain three-dimensional shapes that result in laminar flow and prevent flow separation, and hence result in low drag.

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# Topološko igrišče: frustrirana nematska polja

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Frustracija nematkih faz zaradi kompleksne geometrijske ograditve in možne lastne kiralnosti, omogoča obstoj stabilnih in metastabilnih defektnih struktur v orientacijski urejenosti. Ti sistemi so primeri topološke mehke snovi, za katero je značilna izrazito velika odzivnost na zunanje vplive. Ta lastnost je posebej pomembna za urejanje in samourejanje kompleksnih mehkih snovi, kot tudi za njihovo potencialno uporabo v fotoniki, optiki in senzoriki. V okviru predavanja bom predstavil topološko mehko snov in opisal naše novejšje dosežke, ki lepo ilustrirajo pomen sinergije teorijskih, numeričnih in eksperimentalnih pristopov [1-4].

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# Topological playground: highly constrained nematic fields

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Frustration due to complex geometrical constraints and/or intrinsic chirality leads to stable and metastable complex networks of orientational defects in nematic mesophases. Topological soft matter is the term often used to characterize this kind of materials. They are particularly responsive to external stimuli which is crucial for assembling or self-assembling of complex structures and for possible applications in optics, photonics, and sensorics. In the overview I describe our recent achievements resulting from the synergy of theoretical, numerical, and experimental approaches [1-4].

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# Veliki spektroskopski pregledi zvezd

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Naša Galaksija je ena od tipičnih galaksij v vesolju, obenem pa takorekoč edina, kjer lahko opazujemo posamezne zvezde. Mnoge od njih dosega 50%, nekatere pa celo več kot 90% starosti vesolja. Raziskovanje kemične in dinamične zgodovine zvezd v Galaksiji tako lahko razjasni njen nastanek in razvoj naše galaksije kot tipične galaksije v vesolju. Pristop, ki je alternativa kozmološkemu opazovanju oddaljenih galaksij, je uveljavljen kot lokalna kozmologija ali galaktična arheologija [1]. Skoraj vse fizikalne informacije o zvezdah lahko izvemo le s spektroskopskim opazovanjem, ki pa mora zajeti zelo veliko število zvezd, ki jih vidimo razporejene preko celotnega neba. Zato potrebujemo torej velike spektroskopske preglede neba. Predstavil bom rezultate projekta RAVE [2], ki je najpopolnejši pregled doslej, ter prve rezultate projektov Gaia-ESO [3], Hermes/GALAH [4] in Gaia [5], v katerih sodelujemo.

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# Large spectroscopic stellar surveys

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Our Galaxy is one of the typical galaxies in the Universe and basically the only one where individual stars can be observed. Many of these stars are as old as 50% and some even more than 90% of the age of the Universe. Research of chemical composition and dynamical properties of stars in the Galaxy can therefore reveal the formation and evolution of our galaxy as a typical galaxy in the Universe. The approach which is an alternative to cosmological observations of distant galaxies is known as local cosmology or galactic archaeology [1]. Nearly all physical information on stars can be studied only with spectroscopic observations, which need to include a very large number of stars distributed across the whole sky. So large spectroscopic survey are needed for the task. Results of the RAVE project [2] which is the largest survey so far will be presented, along with the first results of the projects Gaia-ESO [3], Hermes/GALAH [4], and Gaia [5], all of which we are part of.

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