Fakultät für Physik

UNIVERSITÄT DUISBURG ESSEN

Econophysics V:

Credit Risk

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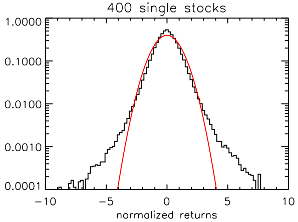
Let's Face Chaos through Nonlinear Dynamics, Maribor 2011

Outline

- Introduction
- Market risk versus Credit risk
- Reduced form models versus Structural models
- Loss distribution
- Numerical simulations and Random matrix approach
- Conclusions: general, present credit crisis

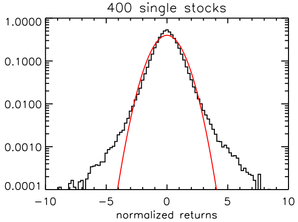
Introduction

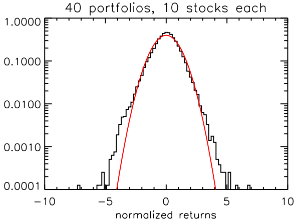
Diversification in a stock portfolio, no correlations



- empirical distribution of normalized returns (400 stocks)

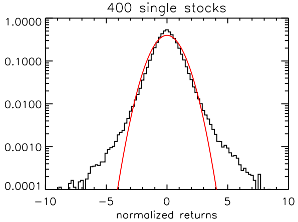
Diversification in a stock portfolio, no correlations

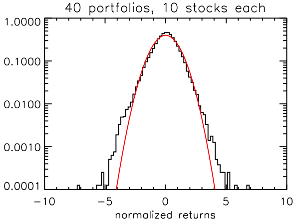




- empirical distribution of normalized returns (400 stocks)
- portfolio: superposition of stocks

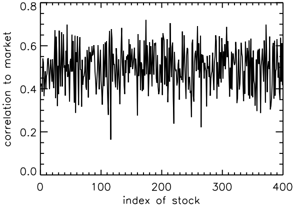
Diversification in a stock portfolio, no correlations

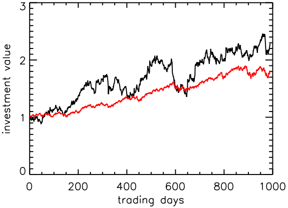




- empirical distribution of normalized returns (400 stocks)
- portfolio: superposition of stocks
- risk reduction by diversification (no correlations yet!):
  returns are more normally distributed,
  market risk reduced by approx. 50 percent

Correlations



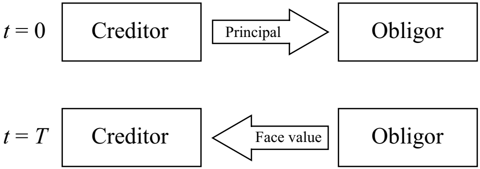


- stocks highly correlated to overall market
- risk reduction by diversification (with correlations):
  unsystematic risk can be removed,
  systematic risk (overall market) remains

Market risk versus Credit risk

What's different for credits?

Zero-coupon bond



- principal: borrowed amount
- face value F:
  borrowed amount + interest + risk compensation
- credit contract with simplest cash-flow

Defaults and Losses

- default occurs if the obligor fails to repay

  $\Rightarrow$ loss between 0 and face value F

- possible losses have to be priced into credit contract
- correlations are important to evaluate the risk of a credit portfolio

Defaults and Losses

- default occurs if the obligor fails to repay

  $\Rightarrow$ loss between 0 and face value F

- possible losses have to be priced into credit contract
- correlations are important to evaluate the risk of a credit portfolio
- statistical modeling needed
- reduced form models versus structural models

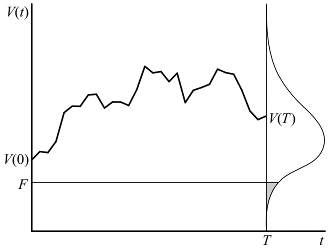
Reduced form models

- macroscopic approach
- different aspects (observables) are modelled independently
  - default events as point process
  - recovery rates modelled independently
  - correlations e.g. as network model

Reduced form models

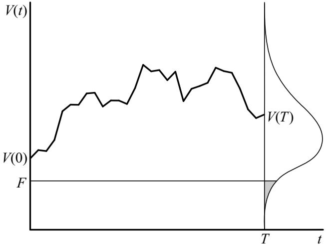
- macroscopic approach
- different aspects (observables) are modelled independently
  - default events as point process
  - recovery rates modelled independently
  - correlations e.g. as network model
- goal: describe empirical statistical properties and market prices for credit products by callibrating with credit products
- problem: the market may be wrong!

Structural models



- microscopic approach
- dynamical description of risk factors $V_k(t),\ k = 1, \ldots, K$
- default occurs if asset value $V_k(T)$ falls below face value F_k
- then the (normalized) loss is $L_k = \frac{F_k - V_k(T)}{F_k}$

Structural models



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- default occurs if asset value $V_k(T)$ falls below face value F_k
- then the (normalized) loss is $L_k = \frac{F_k - V_k(T)}{F_k}$
- e.g. credits with stock portfolio or houses as securities

Modeling credit risk

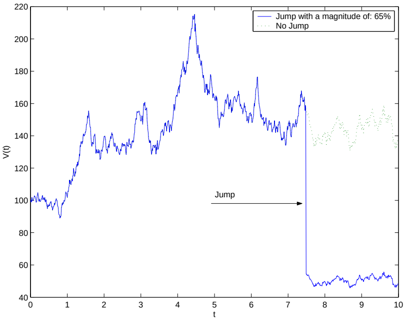
A model with jumps and correlations

$$\frac{\mathrm{d}V_k}{V_k} = \mu_k \,\mathrm{d}t + \sigma_k \varepsilon_k \sqrt{\mathrm{d}t} + \mathrm{d}J_k$$

Geometric Brownian motion with

- deterministic term $\mu_k \,\mathrm{d}t$
- diffusion term $\sigma_k \varepsilon_k \sqrt{\mathrm{d}t}$
- jump term $\mathrm{d}J_k$, governed by a Poisson process
- K risk elements $V_k = V_k(t),\ k = 1, \ldots, K$

Jump process and return distribution



jumps yield heavy tails in the price and return distributions

Jumps as Poisson process

- we model jumps by Poisson process with intensity λ
- probability for n jumps between 0 and t:

$$p_n(t) = \frac{(\lambda t)^n}{n!} \exp(-\lambda t)$$

- largest negative jump: -100% of $V(t)$
- we choose shifted log-normal distribution for jump size Λ

$$\ln(\Lambda + 1) \sim \mathrm{N}(\mu_J + 1, \sigma_J)$$

Correlate K risk elements: one-factor model

- ε_k is random variable for company k
- η is common random variable within one branch
- correlated diffusion, uncorrelated jumps:

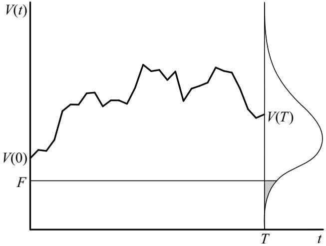
$$\frac{\mathrm{d}V_k}{V_k} = \mu_k \,\mathrm{d}t + \left(\sqrt{1-c}\,\varepsilon_k + \sqrt{c}\,\eta\right)\sigma_k\sqrt{\mathrm{d}t} + \mathrm{d}J_k$$

- add influence of market as a whole

$$\frac{\mathrm{d}V_k}{V_k} = \mu_k \,\mathrm{d}t + \left(\sqrt{1-c}\,\varepsilon_k + \sqrt{c}\,\eta\right)\sigma_k\sqrt{\mathrm{d}t} + \mathrm{d}J_k + \mathrm{d}J_{\mathrm{m}}$$

Loss distribution

Individual losses



- normalized loss: $L_k = \frac{F_k - V_k(T)}{F_k}$
- default probability: $P_{D,k} = \int\limits_0^{F_k} p_k(V_k(T))\,\mathrm{d}V_k(T)$
- truncate distribution $p_k(V_k(T)) \to p_k(L_k)$

Default event

- default indicator

$$I_k = \begin{cases} 1 \quad , & \text{if} \quad V_k(T) < F_k \quad \text{(default)} \\ 0 \quad , & \text{if} \quad V_k(T) > F_k \quad \text{(no default)} \end{cases}$$

- indicator distribution

$$\tilde{p}_k(I_k) = (1 - P_{D,k})\delta(I_k) + P_{D,k}\delta(I_k - 1)$$

Portfolio loss distribution

- portfolio loss: $L - \frac{1}{K} \sum_{k=1}^{K} L_k l_k$
- loss distribution

$$p(L) = \int_{-\infty}^{+\infty} dl_1 \tilde{p}_1(l_1) \cdots \int_{-\infty}^{+\infty} dl_K \tilde{p}_K(l_K) \int_0^1 dL_1 p_1(L_1) \cdots \int_0^1 dL_K p_K(L_K)$$
$$\times \delta \left(L - \frac{1}{K} \sum_{k=1}^{K} L_k l_k \right)$$

- special case $K = 1$ yields: $p(L) = (1 - P_{D,1})\, \delta(L) + P_{D,1} p_1(L)$

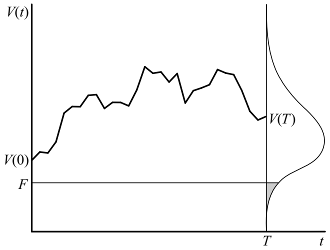
Large portfolios

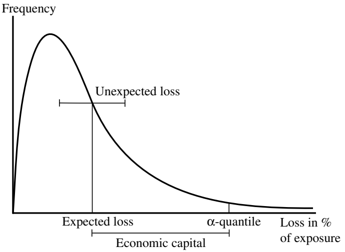
Real portfolios comprise several hundred or more individual contracts $\longrightarrow$ K is large.

Central Limit Theorem: For very large K, portfolio loss distribution $p(L)$ must become Gaussian.

Question: how large is “very large” ?

Distribution of credit losses

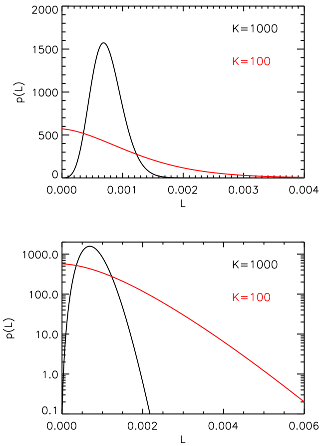




- portfolio loss is arithmetic mean of individual losses
- mean of loss distribution is called expected loss (EL)
- standard deviation is called unexpected loss (UL)
- kurtosis excess (KE) to measure heavy tails: $\gamma_2 = \mu_4/\mu_2^2 - 3$

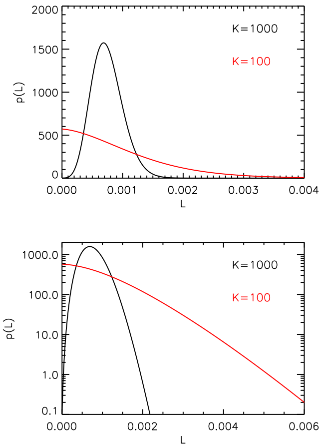
Simplified model — no jumps, no correlations

- homogenous portfolio
- analytical approximations
- check Monte-Carlo results



Simplified model — no jumps, no correlations

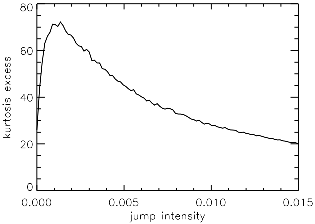
- homogenous portfolio
- analytical approximations
- check Monte-Carlo results
- slow convergence to Gaussian for large portfolio
- $K = 1000$ not yet Gaussian CLT–limit
- kurtosis excess of uncorrelated portfolios scales as $1/K$

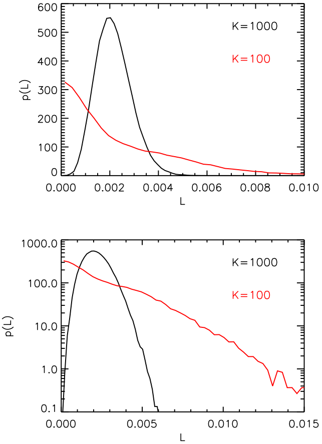


Numerical simulations

Numerical simulations: influence of jumps, no correlations

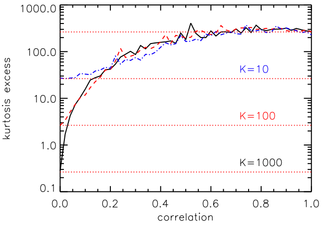
- diffusion and jumps compete
- KE has maximum, but scales as $1/K$

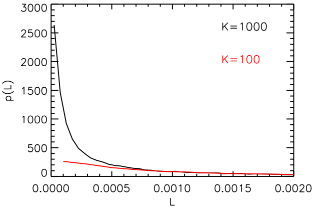


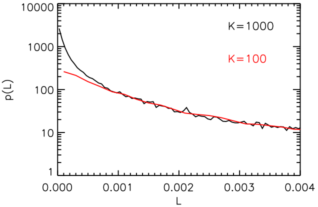


Numerical simulations: influence of correlations, no jumps

- correlation coefficient $c = 0.5$
- transition from uncorrelated to fully correlated



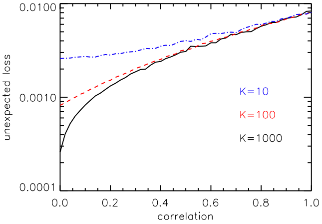


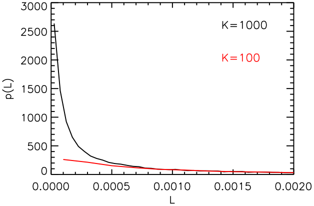


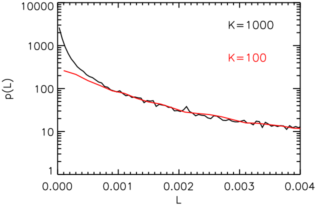
$c = 0.5$

Numerical simulations: influence of correlations, no jumps

- standard deviation decreases
- bad measure for credit risk!
- diversification does not reduce the risk



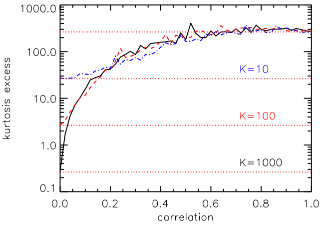


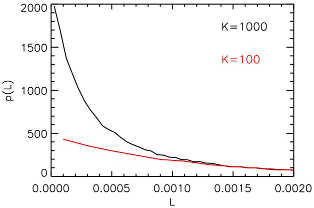


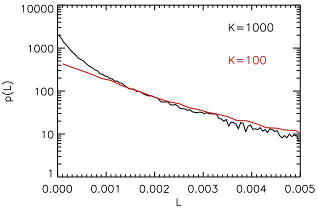
$c = 0.5$

Numerical simulations: influence of correlations, no jumps

- correlation coefficient $c = 0.2$
- transition from uncorrelated to fully correlated



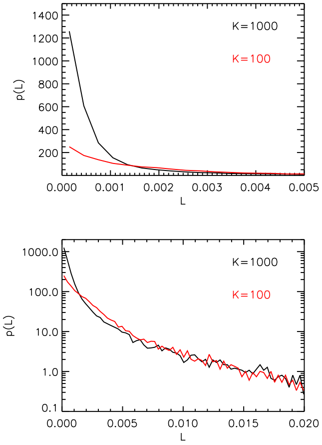




$c = 0.2$

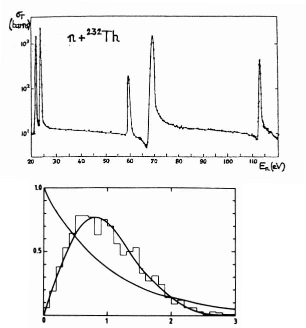
Numerical simulations: jumps and correlations

- correlated jump-diffusion
- one-branch correlations
- $c = 0.5$
- tail behavior stays similar with increasing K



Random matrix approach

Quantum Chaos



statistical nuclear physics

universal in a huge variety of systems: nuclei, atoms, molecules, disordered systems, lattice gauge quantum chromodynamics, elasticity, electrodynamics

"second ergodicity": spectral average = ensemble average

$\longrightarrow$ random matrix theory

Price distribution at maturity

Brownian motion, $V = (V_1(T), \ldots, V_K(T))$, price distribution

$$p^{(\mathrm{mv})}(V, \Sigma) = \frac{1}{\sqrt{2\pi T}^K} \frac{1}{\sqrt{\det \Sigma}} \exp\left(-\frac{1}{2T}(V - \mu T)^\dagger \Sigma^{-1} (V - \mu T)\right)$$

covariance matrix $\Sigma = \sigma W W^\dagger \sigma$ with fixed $\sigma = \mathrm{diag}\,(\sigma_1, \ldots, \sigma_K)$

assume Gaussian distributed correlation matrix $WW^\dagger$
with W rectangular real $K \times N$, variance $1/N$

$$p^{(\mathrm{corr})}(W) = \sqrt{\frac{N}{2\pi}}^{KN} \exp\left(-\frac{N}{2} \mathrm{tr}\, W^\dagger W\right)$$

average correlation is zero

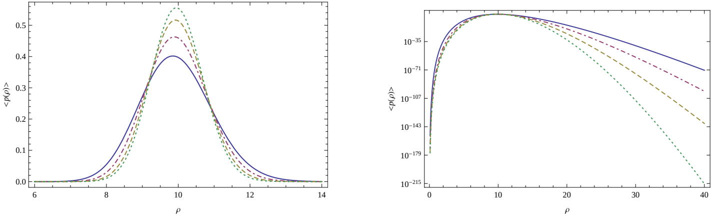
Average price distribution

$$\langle p^{(\mathrm{mv})}(\rho)\rangle = \sqrt{\frac{N}{2\pi T}}^{K} \frac{2^{1-\frac{N}{2}}}{\Gamma(N/2)} \rho^{\frac{N+K-1}{2}} \sqrt{\frac{N}{T}}^{\frac{N-K}{2}} \mathcal{K}_{\frac{N-K}{2}}\left(\rho\sqrt{\frac{N}{T}}\right)$$

with hyperradius $\rho = \sqrt{\sum_{k=1}^{K} \frac{V_k^2(T)}{\sigma_k^2}}$

easily transferred to geometric Brownian motion

Heavy tailed average distribution

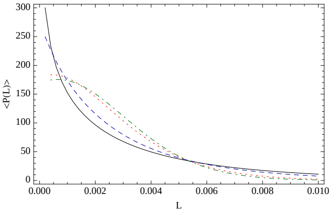


$K = 50$ and $N = K, 2K, 5K, 30K$

N smaller $\longrightarrow$ stronger correlated $\longrightarrow$ heavier tails

Loss distribution — varying correlation strength

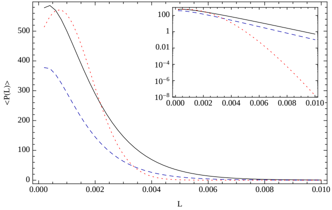
integrate out risk elements, semi–analytical result

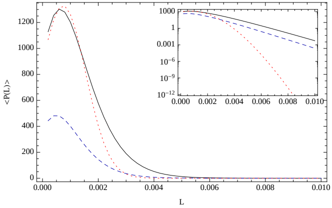


homogeneous portfolio $K = 10$ and $N = K, 2K, 10K, 30K$

also here: stronger correlated $\longrightarrow$ heavier tails

Loss distribution — varying portfilio sizes





homogeneous portfolios $K = 50, 100$, strongly correlated $N = K$

heavy tails robust !

General conclusions

- correlated jumps lead to extremely fat-tailed distribution
- kurtosis excess (KE) scales as $1/K$ for uncorrelated portfolios
- KE does not scale down well for correlated portfolios, even for low correlation coefficients
- correlations of stocks to market movement typically between 0.4 and 0.6
- other scenarios: houses, cars, etc as security for credits
- ensemble average reveals generic features of loss distributions
- lower bound, because average correlation is zero

Conclusions in view of the present credit crisis

- credit contracts with high default probability, e.g. houses as securities
- credit institutes resold the risk of credit portfolios, grouped by credit rating
- lower ratings ⇒ higher risk and higher potential return
- problems:
  - rating agencies rated way too high
  - effect of correlations underestimated
  - benefit of diversification overestimated

R. Schäfer, M. Sjölin, A. Sundin, M. Wolanski and T. Guhr,
Credit Risk - A Structural Model with Jumps and Correlations,
Physica **A383** (2007) 533

M.C. Münnix, R. Schäfer and T. Guhr,
A Random Matrix Approach to Credit Risk,
arXiv:1102.3900

both ranked for several months among the top–ten new credit risk papers on www.defaultrisk.com