# Abstracts of Invited Lectures

**On the Variety of Recurrence Phenomena in Chaotic Dynamics**

**-Revisit to Hamiltonian Chaos from Infinite Ergodicity-**

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Non-stationary recurrence motions are often observed in chaotic systems. Strong intermittency is a typical example which reveals slow dynamics with $1/f^{\nu}$ spectra or slow relaxation of correlation functions (Aizawa,1984, 1989). In non-integrable hamiltonian systems also there appear such long time tails as an universal phenomenon, where it is surmised that sticky regions or stagnant layers surrounding the invariant sets such as KAM tori or Poincaré-Birkhoff tori play an essential role to induce a kind of strong intermittency with non-stationarity of phase-space trajectories (Aizawa *et al.*, 1989). Detailed structures and the recurrent mechanisms in the stagnant layers have not yet been elucidated clearly in generic cases not only of high dimensional systems but also of small degrees systems. In this report, we consider several cases which display essentially different scaling laws in the recurrent time distribution. Especially our main concerns will be paid toward the non-stationary recurrence properties in relation to the infinite measure ergodicity.

In the first part we discuss the phenomenology in high dimensional hamiltonian systems with many degrees of freedom; lattice vibrations and cluster formation, where it is emphasized that the distribution of survival times obeys the Log-Weibull law, and that the universal law is consistent with the Nekhoroshev's estimation of the characteristic time in Arnold diffusion. Another type of recurrent feature with inverse power law is discussed by carrying out with the Mixmaster universe model of Bianchi type IX, where the infinite measure ergodicity is proved in a lower dimensional subdynamics though the Mixmaster model is totally a hamiltonian system with three degrees of freedom (Aizawa, 1997). The inverse power laws in recurrence time distributions are exactly shown in other two dimensional mappings (modified Cat map, multi-baker's map, and Mushroom billiard, etc) (Miyaguchi, 2007).

In the second part of my talk, we discuss the re-injection mechanism in the stagnant layers by use of several one-dimensional maps (modified Bernoulli map, Log-Weibull map, and Ant-lion map (infinite modal singular map)) which reveal non-stationary long time tails and infinite measure ergodicity. Those one-dimensional systems are not exact reduction from the corresponding high dimensional hamiltonian flows, but we expect that the same type of recurrent mechanism might be derived in high dimension cases. We discuss the origin of the universal scaling relations for various classes of recurrent phenomenon. Some theoretical and numerical results obtained from the renewal theories and the large deviation theories are introduced, and the interrelation between the infinite ergodicity and the non-stationary recurrence phenomena is explained. Especially, it is emphasized that the correlation functions and the power spectral density are random variables obeying universal distributions in the non-stationary regime (Akimoto, 2007). From these results quite similar to the hamiltionian chaos, it is conjectured that the infinite measure ergodicity might be hidden behind the multi-ergodic features of the stagnant layers in hamiltionian chaos (Aizawa, 2005).

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# Dynamical tunneling in systems with a mixed phase space

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Typical Hamiltonian systems have a mixed phase space consisting of regular islands, which are dynamically separated from the chaotic sea. While classically no transition between those regions is possible, they are quantum mechanically coupled by the process of dynamical tunneling. We derive theoretical predictions for dynamical tunneling rates, which describe the decay of regular states to the chaotic sea. Using an approach based on the introduction of a fictitious integrable system, agreement with numerical data is found for 1D kicked systems [1] and the mushroom billiard [2]. Finally, we will discuss the relevance of a precise knowledge of dynamical tunneling rates for flooding of regular islands [3,4], transport in rough nano-wires [5], and spectral statistics in systems with a mixed phase space.

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# Applications of the GALI Method to Localization in Nonlinear Systems

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We investigate localization phenomena and stability properties of quasiperiodic oscillations in $N$ degree of freedom Hamiltonian systems and $N$ coupled symplectic maps. In particular, we study an example of a parametrically driven Hamiltonian lattice with only quartic coupling terms and a system of $N$ coupled standard maps. We explore their dynamics using the Generalized Alignment Index (GALI), which constitutes a recently developed numerical method for detecting chaotic orbits in many dimensions, estimating the dimensionality of quasiperiodic tori and predicting slow diffusion in a faster and more reliable way than most other approaches known to date.

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# The Stability of Vertical Motion in the N-body Sitnikov Problem

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We present results about the stability, bifurcations and families of 3D-periodic orbits of the Sitnikov motions in the case of the restricted N-body problem. Here we consider $\nu$=N-1 equal mass primary bodies which rotate on a circle and the Nth body moves perpendicular to the plane of the primaries. We extend previous work on the four-body problem to the N-body problem, for N=5, 9, 15 and 25. We found that the Sitnikov family, in all cases under consideration, has only one stability interval. For N=5, 9, 15, 25 we have 14, 16, 18, 20 correspondingly, critical Sitnikov periodic orbits from which 3D-families (no longer rectilinear) bifurcate. We have also studied the fascinating question of the extent of bounded dynamics away from the z-axis, taking initial conditions on x,y planes, at constant $z(0) = z_0$ values, where $z_0$ lies within the interval of stable rectilinear motions. We also performed a similar study of the dynamics near some members of 3D families of periodic solutions and found, on suitably chosen Poincaré $(x, \dot{x})$ surfaces, "islands" of ordered motion, while further away from them most orbits become chaotic and eventually escape to infinity.

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# Dynamical Properties and Synchronization of Complex Nonlinear Equations for Detuned Lasers

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We study the dynamics and synchronization properties of a system of complex nonlinear equations describing detuned lasers. These equations possess a whole circle of fixed points, while the corresponding real variable equations have only isolated fixed points. Studying the stability of their equilibrium points, we determine conditions under which the complex equations have positive, negative or zero Lyapunov exponents and chaotic, quasiperiodic or periodic attractors for a wide range of parameter values. We investigate the synchronization of chaotic solutions of our detuned laser system, using as a drive a similar set of equations and applying the method of global synchronization. We find attractors whose 3–dimensional projection is not at all similar to the well–known shape of the (real) Lorenz attractor. Finally, we apply complex periodic driving to the electric field equation and show that the model can exhibit a transition from chaotic to quasiperiodic oscillations. This leads us to the discovery of an exact periodic solution, whose amplitude and frequency depend on the parameters of the system. Since this solution is stable for a wide range of parameter values, it may be used to control the system by entraining it with the applied periodic forcing.

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# Classical and quantum transport: from Fourier law to thermoelectric efficiency

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The understanding of the underlying dynamical mechanisms which determines the macroscopic laws of heat conduction is a long standing task of non-equilibrium statistical mechanics. A better understanding of such mechanism may also lead to potentially interesting applications based on the possibility to control the heat flow. Of particular interest is the problem, almost completely unexplored, of the derivation of Fourier law from quantum dynamics. To this end we discuss heat transport in a model of a quantum interacting spin chain and we provide clear numerical evidence that Fourier law sets in above the transition to quantum chaos. Finally we consider the transport of particles and heat in open classical ergodic billiards. We show that thermoelectric efficiency can approach the Carnot limit for sufficiently complex charge carrier molecules.

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# Lecture 1: Hopf's dynamical vision of turbulence

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As a turbulent flow evolves, every so often we catch a glimpse of a familiar pattern. For any finite spatial resolution, the system follows approximately for a finite time a pattern belonging to a finite alphabet of admissible patterns. In "Hopf's vision of turbulence," the long term turbulent dynamics is a walk through the space of such unstable patterns.

In this 3-lecture mini-course we start with a recapitulation of basic notions of dynamics; flows, maps, local linear stability, heteroclinic connections, qualitative dynamics of stretching and mixing and symbolic dynamics.

In lecture 2 we discuss the discrete and continuous symmetries of 1- and 3-dimensional fluid flows, and in lecture 3 we illustrate the dynamical approach by moderate *Re* boundary sheer turbulence in the plane Couette flow.

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# Lecture 2: Symmetries and dynamics

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Dynamical systems often come equipped with discrete symmetries, such as the reflection symmetries of various potentials. Symmetries simplify the dynamics in a rather beautiful way: If dynamics is invariant under a set of discrete symmetries $G$, the state space $\mathcal{M}$ is *tiled* by a set of symmetry-related tiles, and the dynamics can be reduced to dynamics within one such tile, the *fundamental domain* $\mathcal{M}/G$. If the symmetry is continuous the dynamics is reduced to a lower-dimensional desymmetrized system $\mathcal{M}/G$, with "ignorable" coordinates eliminated (but not forgotten). In either case families of symmetry-related full state space cycles are replaced by fewer and often much shorter "relative" cycles. In presence of a symmetry the notion of a prime periodic orbit has to be reexamined: it is replaced by the notion of a relative periodic orbit, the shortest segment of the full state space cycle which tiles the cycle under the action of the group. Furthermore, the group operations that relate distinct tiles do double duty as letters of an alphabet which assigns symbolic itineraries to trajectories.

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# Lecture 3: Geometry of boundary shear turbulence - a stroll through 61,506 dimensions

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In the world of moderate Reynolds number, everyday turbulence of fluids flowing across planes and down pipes a velvet revolution is taking place. Experiments are almost as detailed as the numerical simulations, DNS is yielding exact numerical solutions that one dared not dream about a decade ago, and dynamical systems visualization of turbulent fluid's state space geometry is unexpectedly elegant.

We shall take you on a tour of this newly breached, hitherto inaccessible territory. Mastery of fluid mechanics is no prerequisite, and perhaps a hindrance: the lecture is aimed at anyone who had ever wondered why - if no cloud is ever seen twice - we know a cloud when we see one? And how do we turn that into mathematics?

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# Dynamics of a large population of coupled active and inactive oscillators

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Coupled nonlinear oscillators governed by dissipative dynamics have been studied extensively in the past decades. Such a system is composed of either limit-cycle oscillators or chaotic oscillators. In particular, much attention has been paid to a variety of interesting phenomena observed in a large ensemble of such oscillators, e.g. synchronization transition and clustering. Results of these studies are of significance in diverse areas of science and technology.

However, there is one point overlooked in such studies, which is the fact that real coupled oscillators, like any other systems, suffer from some kind of deterioration from the beginning or as time passes. Motivated by this, we here consider the effect of "bad components" on the behavior of a population of coupled oscillators, where a "bad component" means an oscillator which has lost the ability of performing self-sustained oscillation. Such a component of the population will be called an "inactive" oscillator, while a component keeping that ability an "active" one. More specifically, we examine what happens in such a system as the ratio of inactive elements $p$ as well as the coupling strength $K$ are varied. This problem is important, for example, in understanding the robustness of diverse biological rhythms and in technological contexts, where no system is allowed to be fragile to defects.

After an introduction, we start from some mathematical models of globally and diffusively coupled oscillators which are either periodic or chaotic. Here, we encounter such phenomena as aging transition and desynchronization or clustering. We then proceed to the case of locally coupled oscillators to examine what new features the locality of coupling brings to the system.

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# Entropy, Information and Dynamical Systems: mathematical results and applications

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*The discovery that the amount of information in a message (or in any other structure) can be objectively measured was certainly one of the major scientific achievements of the 20th century* (Peter Grassberger)

We shall first review the very basic definition of the *information theoretic entropy* for symbolic sequences, discussing its main mathematical properties and its close relationship to thermodinamic entropy. Then we like to discuss the importance of information entropy for chaotic systems and to mention the role that entropy plays in Bayesan inference (*maximum entropy principle*).

As we will see, mostly through simple examples, estimating the entropy of a message (text document, picture, piece of music or biological signal) is quite important because it gives a measure of its *compressibility*, i.e. the optimal achievement for any possible compression algorithm. The very basic definitions and classical results of coding theory will give us the opportunity to understand in simple set up the exact relationship between entropy and compressibility (*Shannon's (noiseless) coding theorem*).

We then turn to *relative entropy* (also called *Kullback-Leibler divergence*) that allow us to measure statistical differences between two distinct sources of information. Our main objective is to discuss its main mathematical properties and how this quantity can be approximated in practice through a suitable use of everyday (e.g. *winzip* or *gzip*) compression algorithm (*Merhav-Ziv's Theorem*).

After a brief presentation of other recent entropy indicators, together with their applications to sequences generated by chaotic dynamical systems, we conclude discussing how these ideas can be applied to more concrete situations, such as the problem of *authorship attribution* for literary text and automatic classification of cardiac signals.

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# Multiplanetary Solar Systems – A Challenge for Astronomy

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We shall review the structure of the known solar systems with more than one planet. To be able to compare our own system with the known more than 200 extrasolar planetary system (EPS) we discuss the dynamical structure of our planets with this interesting separation in an inner system with the four terrestrial planets and the outer one with the four gasgiants. We also show the new results concerning the longterm stability (unstability?) over billions of years.

We then discuss the most interesting EPS which are stable although some of them they suffer from quite large eccentric orbits. In addition we show results of the possible additional – namely terrestrial – planets which could be interesting from the point of view of searching for other earthlike planets.

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# Multiple ionization in strong laser fields

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Strong field multiple ionization has puzzled scientists, among other things, because of the strong correlations among outgoing electrons: their momenta parallel to the polarization axis of the field are very similar. In an effort to explain this process, we have studied the classical dynamics of two electrons escaping from an attracting nucleus. This helps to understand the origin of the correlations and suggests a simplified model that captures the essential process and can be simulated in full quantum calculations.

We find that the repulsion between electrons amplifies deviations from a symmetric escape and thereby enforces the correlates escape between the electrons. This observation can be exploited to describe many of the observed properties, to derive a nonlinear threshold law near the onset of double ionization, and to develop a simple 1+1-dimensional model for effective classical and quantum mechanical simulations. Quantum simulations of this model show good agreement with the classical model, but also reveal fluctuations, which semiclassically can be understood as interferences from multiple paths leading to double ionization.

The dynamics of the correlations can easily be extended to simultaneous escape of more than two electrons, and to multiple ionization in molecules.

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# Reversibility in many body physics

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The classical equations of motion for particles are reversible in that the trajectories are retraced if the velocities and the direction of time are reversed. This holds true independent of whether the motion is integrable or chaotic. The difference is noticed, however, in the presence of noise or stochastic forces: then the variances increase algebraically in the case of an integrable motion and exponentially in the case of a chaotic motion.

GI Taylor has demonstrated the reversibility in the integrable case for a viscous fluid between rotating concentric cylinders, Chaiken et al has demonstrated the increased stretching in the case of chaotic motions. A classical analysis is given in Eckhardt (2003).

In a twist to these experiments, Pine et al have recently replaced the colored fluid in these experiments by micron-sized particles. Because of the small Reynolds numbers involved the dynamics is overdamped and the dynamics very much dissipative. They noticed that after several oscillations the system self-organizes into one of two kinds of states: (i) for small amplitudes the particles arrange themselves such that they avoid interactions. In this state the dynamics is reversible. (ii) for larger amplitudes the particles cannot avoid interations and the dynamics is irreversible.

We will discuss the equations of motion for a classical particle in a liquid, present simulations that show the kinds of transitions between the states and discuss evidence for the phase transitions and its critical exponents.

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for movies, see http://www.physics.nyu.edu/pine/research/hydroreverse.html

# Periodic orbits, localization in normal mode space, and the Fermi-Pasta-Ulam problem

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In 1955 Fermi, Pasta and Ulam (FPU) reported on the nonequipartition of a nonlinear atomic chain, with initially one normal mode excited. Modern computational studies show, that on a first, relatively short, time scale the energy is distributed among a few neighbouring modes in modal space, with more distant modes being exponentially weakly excited - i.e., one observes localization in normal mode space. On a much larger second time scale (which was not reachable wih the MANIAC I), the tail modes are slowly growing, and finally the system does equilibrate. Despite its strong impact on nonlinear dynamics and statistical physics, the paradox remained essentially unexplained for decades.

Recent studies show that the model allows for exact time-periodic solutions (q-breathers), which are exponentially localized in the space of normal modes. The trajectory initially computed by FPU is a slight perturbation away from an exact q-breather orbit. Consequently most of the key observations related to the FPU problem (localization of energy in normal mode space for long times, recurrence on relatively short times, system size and energy thresholds) are captured by the properties of q-breathers and the phase space flow nearby. The underlying concept is much more general, and can be easily extended to two-and three-dimensional finite lattices. In particular, localization properties of q-breathers are shown to depend on intensive control parameters (energy density, wave vectors) ONLY.

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# Localization Versus Delocalization in Nonlinear Disordered Systems

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Linear disordered systems allow for Anderson localization. Then all eigenstates are localized, and an initially localized wave packet will not spread beyond the localization volume of the linear system. Adding nonlinearities leads to an interaction of eigenstates. The question is therefore, whether the wave packet will spread or stay localized. I will show that the wave packet can spread in three different regimes. All of them show up with subdiffusion, while some allow also for partial localization due to selftrapping. The subdiffusive spreading is universal and characterized by the second moment growing algebraically in time, with exponent 1/3 for one-dimensional systems with cubic nonlinearity. It is due to a finite number of modes which stay resonant and are responsible for weak chaos inside the packet.

I will generalize to other types of nonlinearity, higher spatial dimensions, address the spreading in the presence of a nonzero thermal background, and the corresponding quantum case.

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# Self-Organized Criticality in Neuronal Systems*

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Self-organized criticality is one of the key concepts to describe the emergence of complexity in natural systems. The concept asserts that a system self-organizes into a critical state where system observables are distributed according to a power law. It has long been speculated that this phenomenon might also show up in neuronal networks, but so far no genuinely neuronal model has been shown to exhibit full self-organized criticality.

Here we consider a network of integrate-and-fire neurons with depressive dynamical synapses, i.e. where the synaptic coupling exhibits fatigue under repeated presynaptic firing [1]. We find self-organized critical avalanches and show that in a range of interaction parameters this adaptation mechanism drives the network into a self-organized critical regime by adjusting the average coupling strengths to a critical value. We derive an analytic expression for the mean synaptic strengths and the average inter-spike intervals in a mean-field approach. These mean values obey a self-consistency equation which allows us to characterize the self-organization mechanism. Our theory explains recent experimental results, where neuronal avalanches were observed in multi-electrode recordings of cortical slice cultures [2].

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$*$ work in collaboration with A. Levina and M. Herrmann

# Levydemics*

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The efficiency of epidemic modelling and forecasts has suffered in the past from a poor description of the spatial dynamics. Accurate models are needed e.g. to test potential strategies to control the spread of an epidemic. While the local infection dynamics is well understood for many diseases, very little was known about the statistical laws by which humans and their germs disperse. We have tried to improve these models by introducing Lèvy processes, which allow for superdiffusive spatial dynamics and have carried out an experiment to verify their applicability.

How can we obtain reliable information on travelling statistics, if people can travel using very different means of transportation from bikes to planes? We have studied this problem empirically using the dispersal of dollar bills as a proxy. The time dependent probability density obtained in this way exhibits pronounced spatiotemporal scaling and anomalous diffusion, which mathematically are described very accurately by our model in terms of a bifractional diffusion equation with few parameters.

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∗ work in collaboration with D. Brockmann and L. Hufnagel

# Thermal convection beyond the Oberbeck-Boussinesq simplification
# I
# The phenomenon

**Siegfried Grossmann**

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These three lectures are based on joint work with the following co-authors:
**Detlef Lohse, Francisco Fontenele Araujo, Guenter Ahlers, Eric Brown, Denis Funfschilling, Kazuyasu Sugiyama, Enrico Calzavarini, and Alexander Esser**

An introductory presentation of the basic physics of thermal convection in Rayleigh-Bénard geometry between a warmer bottom plate and a colder top plate is given. The heat current $Q$ or Nusselt number $Nu = Q/(\kappa \Delta L^{-1})$ and the amplitude $U$ or Reynolds number $UL/\nu$ of the convective flow are considered as functions of the external control parameters Rayleigh number $Ra = \beta g L^3 \Delta/(\nu\kappa)$ and Prandtl number $Pr = \nu/\kappa$. Recent experimental progress is discussed in terms of a unified theory (GL 2000, and subsequent papers), based on the exact equations of motion.

Particular emphasis is put on a currently explored phase space range of very small Pr numbers (cf. Grossmann & Lohse 2008). It turns out to be strongly turbulent but rather inefficient in heat transport: though $Re$ increases considerably, the Nusselt number decreases, $Nu \to 1$, i.e., the heat transport becomes mainly conductive.

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# Thermal convection beyond the Oberbeck-Boussinesq simplification II
# Physical signatures of non-OB

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An experimental puzzle, measuring different $Nu$ versus $Ra$ for seemingly the same systems prompted the discussion how to eliminate systematic errors: side wall leakage, plate resistance corrections, or deviations from the Oberbeck-Boussinesq approximation. Significant recent progress has in particular been obtained in measuring and interpreting the signatures of thermal convection, if the temperature dependence of the material parameters is taken into account, so called non-Oberbeck-Boussinesq (NOB) effects. This turns out to mainly be a performance of the boundary layers, as was first shown for water and glycerol (cf. Ahlers 2006, Sugiyama 2007). Another interesting example is the RB convection near the critical point of ethane, which gives some surprising insight into the mechanism leading to NOB effects originating in the strong temperature dependence of the thermal expansion coefficient $\beta(T)$ and/or the heat capacity $c_p(T)$, which mediate the thermal driving of heat convection (cf. Ahlers 2007, 2008).

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# Convection beyond the Oberbeck-Boussinesq simplification III Driving by shear

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This third lecture is devoted to recent promising results to explain the momentum (instead of heat) transport in shear driven flows, making use of similar, successful ideas as have been presented in the first two lectures for the heat transport in thermal convection. The relevant systems are Taylor-Couette flow between independently rotating concentric cylinders and turbulent flow through pipes, having angular momentum or axial momentum transport, respectively. At first also for the momentum transport an introduction into the basic physics of shear driven flows based on the Navier-Stokes equation is offered, then the relevant global relations are derived, and finally comparison between theory and the data is provided, which seems quite promising. It turns out that these shear driven flows are non-Oberbeck-Boussinesq-like from the very beginning, although viscosity $\nu$ is a constant through the liquid here. In contrast, in the shear driven Taylor-Couette system the NOB-like effects originate from the radius dependence of the driving centrifugal force which is relevant here instead of the temperature dependence of the material parameters in thermal flow.

The series of lectures is closed by drawing attention to some most urgent but unsolved open questions in the field, namely plume formation and time dependent boundary layer separation in RB thermal convection, taking "non-Oberbeck-Boussinesqness" into better account for the torque-scaling in Taylor-Couette flow, and extending to the large gap case of TC flow.

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# Ergodicity and orbit bunching: Universal features of chaos and their quantum signatures

**Alexander Altland, Petr Braun, Fritz Haake, Stefan Heusler, Sebastian Müller**

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Long periodic orbits of fully chaotic dynamics tend to fill the energy shell with uniform density. As a quantitative expression of that well-known fact the sum rule of Hannay and Ozorio de Almeida will be discussed.

More recently, it was discovered that long periodic orbits in chaotic systems do not arise as mutually independent individuals but rather in closely packed bunches. Bunches owe their existence to the fact that each long orbit displays self-encounters in configuration space where two or more orbit stretches run close together. Along each orbit, encounter stretches alternate with "links". Different orbits in a bunch are nearly identical in the links; these links are differently connected by the encounter stretches. I shall reveal this bunching phenomenon as due to a certain exponential stability of the Hamiltonian boundary-value problem which in turn is equivalent to the exponential instability of the initial-value problem of classical mechanics.

Quantum signatures of bunches arise in the semiclassical limit, for all quantities that can be written as sums of Feynman amplitudes of orbits. When a bunch is so closely packed that the orbit-to-orbit action differences become of the order of Planck's constant the Feynman amplitudes interfere constructively. In this way, universal fluctuations in energy spectra or of transport through mesoscopic conductors arise.

Random-matrix theory (RMT) provides a phenomenological description of universal spectral fluctuations, by replacing individual Hamiltonians with random matrices and calculating useful indicators (spacing distribution of energy levels, spectral form factor, ...) as averages over suitable ensembles of random matrices. Such averages often give simple analytic results. I shall illustrate the fidelity of chaotic dynamics to RMT with a few examples.

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# Semiclassical theory of universal spectral fluctuations for chaotic dynamics

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Gutzwiller's periodic-orbit theory can be used to semiclassically represent the density of energy levels as a formal sum of contributions of periodic orbits. Similarly, the spectral determinant can be approximated as a sum over sets of periodic orbits, the so-called pseudo-orbits. All these sums are divergent, due to the exponential proliferation of periodic orbits with growing period. Only certain correlation functions of such sums can exist and actually be calculated.

This lecture will outline how the two-point correlator of the level density has recently been determined successfully for individual chaotic dynamics, in agreement with the ensemble averages of random-matrix theory. The correlator results as a sum of a non-oscillatory and an oscillatory term; for the Fourier transform with respect to the energy, the "spectral form factor", the two terms mentioned respectively yield the behavior for times up to the Heisenberg time ($0 < t \leq T_H$) and larger ($T_H < t$). The two "terms" of the correlator actually arise as asymptotic series in inverse powers of an energy offset, up to an oscillating exponential factor for the oscillatory term.

For the non-oscillatory part of the two-point correlator one may start from the product of two Gutzwiller sums for the level density. The appropriate starting point for capturing the oscillatory part is a certain generating function, a multiplicative combination of four spectral determinants or their pseudo-orbit representations.

Berry's diagonal approximation involves pairs of identical orbits and gives the leading terms of the asymptotic series involved. Higher-order corrections involve bunches of orbits that differ only in reconnections within close self-encounters.

The contributing orbit bunches look like Feynman diagrams, if close self-encounters are associated with vertices and the intervening links with propagator lines. The resemblance is not at all fortuitous. If the ensemble averages of RMT are implemented perturbatively, within the so-called nonlinear sigma model, Feynman diagrams do uniquely characterize the resulting perturbation series. Each orbit bunch contributing in the semiclassical theory thus turns out one-to-one with a Feynman diagram of RMT.

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# Quantum measurement without Schrödinger cat states

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A quantum measurement involves an object $\mathcal{S}$ and an apparatus; the latter may be idealized to a single-freedom pointer $\mathcal{P}$ interacting with a many-body bath $\mathcal{B}$. The interactions within the three-partite system $\mathcal{S}\oplus\mathcal{P}\oplus\mathcal{B}$ must (i) entangle $\mathcal{S}$ and $\mathcal{P}$ (associating different eigenvalues of the measured object observable with macroscopically distinct pointer displacements) and (ii) decohere the macroscopically distinct pointer states through the action of $\mathcal{B}$.

A class of models will be presented which display the behavior just outlined. In particular, emergence and decoherence of distinct pointer displacements can be allowed to proceed simultaneously such that a mixture of macroscopically distinct states arises directly, without any intermediate macroscopic superposition.

Special models involving harmonic oscillators allow for rigorous solutions of the Schrödinger equation of $\mathcal{S}\oplus\mathcal{P}\oplus\mathcal{B}$. A much wider model class is amenable to explicit solution in the limit where the time scales for emergence and decoherence of macroscopically distinct states are small compared to the characteristic times of the free motions of $\mathcal{S}, \mathcal{P}$ or even $\mathcal{S}, \mathcal{P}, \mathcal{B}$.

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# Wave Chaos in Rotating Optical Microcavities

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The Sagnac effect is the phase difference between two counter-propagating laser beams in rotating resonators, originally introduced by Sagnac in 1913. It has become the basis for the operation of the optical gyroscopes such as ring laser gyroscopes and fiber optic gyroscopes. These optical gyroscopes are normally used in airplanes, rockets, and ships etc. since they are the most precise rotation velocity sensors among any other types of gyroscopes.

The Sagnac effect had been theoretically derived for the slender waveguides like optical fibers or the ring cavities composed of more than three mirrors by assuming that the light propagates one-dimensionally and the wavelength of the light is much shorter than the sizes of the cavities. However, the sizes of the resonant cavities can be reduced to the order of the wavelength by modern semiconductor technologies. The conventional description of the Sagnac effect is not applicable to such small resonant microcavities. Especially, the resonance wave functions are standing waves which can never be represented by the superposition of counter-propagating waves, while the assumptions of the existence of CW and CCW waves plays the most important role for the conventional theory of the Sagnac effect.

By using perturbation theory typically used in quantum mechanics, we show that the Sagnac effect can also be observed even in resonant microcavities if the angular velocity of the cavity is larger than a certain threshold where the standing wave resonance function changes into the rotating wave. For a quadrupole cavity, it is not assumed that the CW and CCW waves exist in the cavity, but the pair of the counter-propagating waves is automatically produced by mixing the nearly degenerate resonance wave functions due to rotation of the cavity.

We also show that the degenerate eigen-frequency corresponding to the wave-chaotic cavity-mode of the non-rotaing cavity splits into two frequencies and their difference is proportional to the rotation rate although the splitting cavity-modes are still wave-chaotic and do not have any corresponding CW and CCW propagating modes as well as ray-dynamical counterparts, which cannot be explained by the conventional Sagnac effect.

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# Quantum Dynamical Semigroup Generators

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We discuss a construction of *quantum dynamical semigroup*, and show
(a) derivation of its *generator* from *operator stochastic differential equation*, (b) relation between *dissipation* and *contraction* of the semigroup and (c) *quantum detailed balance*.

## 1. Quantum stochastic differential equation

This is a quantum (i.e. noncommutative) analogue of ordinary stochastic differential equation which was initiated by K.Itô[2], and can be formulated by using the so-called *stochastic calculus* in parallel with the commutative framework.

$$Y \circ dX = Y \cdot dX + \frac{1}{2} dY DX \quad \circ dX \cdot Y + \frac{1}{2} dX dY \tag{1}$$

where the former and the latter expressions must be distinguished for the reason of noncommutativity $Y \circ dX \neq \circ(dX)Y$. Also,

$$Z \circ (Y \circ dX) = ZY \circ dX \tag{2}$$

and

$$df(X) = \frac{df}{dX} \circ dX = \frac{df}{dX} dX + \frac{1}{2} d^2 f(X). \tag{3}$$

The simbol $\circ$ was introduced by Itô [2] for distinguishing between the two Fokker-Planck equations of Itô and Stratonovich(see [1]).

## 2. Stochastic Schroedinger and stochastic Heisenberg equation

We show a more concrete incorporation of the stochastic element into the ordinary quantum dynamics: our purpose is to ellucidate how statistical feature *dissipativeness* can be formulated on the basis of three rules summarized by eq.(1), eq.(2) and eq.(3). The result can be shown in terms of *quantum dynamical semigroup* and its generator as follows.

$$X_t = \Lambda_t X \equiv e^{tL} X_{t=0} \tag{4}$$

where $\Lambda_t$ is a *superoperator* acting on the space of operators for a given quantum system.
In the above, the superoperator $L$ in $\Lambda_t$ is called *generator* of the (quantum) dynamical semigroup.

## 3. Condition of detailed balance

Let us recall the classical description of *detailed balance condition*[9] expressed as

$$\Sigma_l \frac{\partial a_{il}^{-1} b_l}{\partial x_j} = \Sigma_l \partial a_{jl}^{-1} b_l \partial x_i, \tag{5}$$

where

$$\frac{\partial p(t,x)}{\partial t} - \frac{\partial}{\partial x_i}(-b_i(x) p(t,x) p(t,x) + \frac{1}{2} \frac{\partial}{\partial x_j}(a_{ij}(x) \frac{\partial}{\partial x_i} a_{ij}(x) p(t,x) \tag{6}$$

in terms of *drift vector* $b_i(x)$ and *diffusion tensor* element $a_{ij}(x) (= a_{ji}(x))$. Then, the detailed balance condition is given by eq.(5) [9].

A possible quantum mechanical detailed balance condition can be formulated based on (i) existence of the equilibrium density operator $\omega$ which is unique (ii) the set of unitary time evolutions $\Sigma_t \equiv \{U_t; -\infty < t < \infty\}$ commutes with the semigroup generator $L$ defined in eq.(4).

The details of the above feature will be presented.

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# Asymmetric Heat Conduction in Nonlinear Systems

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Heat conduction is an old yet important problem. Since Fourier introduced the law bearing his name two hundred years ago, a first-principle derivation of this law from statistical mechanics is still lacking. Worse still, the validity of this law in low dimensions, and the necessary and sufficient conditions for its validity are still far from clear. In this talk I'll give a review of recent works done on this subject. I'll also report our latest work on asymmetric heat conduction in nonlinear systems. The study of heat condution is not only of theoretical interest but also of practical interest. The study of electric conduction has led to the invention of such important electric devices such as electric diodes and transistors. The study of heat conduction may also lead to the invention of thermal diodes and transistors in the future.

# Semiclassical assignment of highly excited molecular vibrations

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We present a method to treat highly excited molecular vibrations of small polyatomic molecules. We start from an algebraic Hamiltonian either fitted to the experimental spectrum or constructed from a given potential surface. Its classical counterpart $H$ has several very favorable properties: It has a natural decomposition into an integrable $H_0$ and resonant interactions $W$. The whole Hamiltonian is automatically given in the action/angle variables belonging to $H_0$. The configuration space of the system is the angle space. Polyad type conserved quantities allow a reduction of the number of degres of freedom.

If some of the modes are involved in several independent resonances, then the system is nonintegrable and in many cases shows chaos in large parts of the classical phase space.

First we study the various coupling schemes possible by the functional form of $W$ and the corresponding organization centers in configuration space. Then we construct a semiclassical representation on the configuration space of the eigenstates of the quantum problem and compare them with the organization centers of the various coupling schemes. For the large majority of states we thereby see to which coupling scheme they belong. By counting nodal patterns and phase advances of the semiclassical wave functions we get quantum numbers relative to the respective organization center. As principal example DCO is used. Other examples are mentioned briefly.

The success of this method for systems with a complicated spectrum and with classical chaos poses interesting questions on quantum chaology. Finally we make some suggestions for a possible answer of this question.

# Spatial network representation of complex living tissues

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Complex network theory has recently attracted large attention across several scientific disciplines as it can be used to describe very diverse systems from the network perspective. In this approach the elements of the system are represented by vertices and interactions between the elements by edges or links connecting the vertices. We shall first review the basic elements of network theory focusing on the scale-free networks and spatially embedded scale-free networks in particular. We shall introduce and discuss some of the algorithms for constructing networks with scale-free architecture.

Following this, we will present a network approach to explore the relationship between structure and function in biological systems, concentrating on the case of pancreatic islets of Langerhans. We will describe a structure of an intact living islet of beta cells as a scale-free complex system using a fitness network model embedded in Euclidean space. The connectivity between two beta cells in the model is the function of the spatial localization and conductance between these cells. Both quantities have been experimentally determined in intact pancreatic islets. Several computed properties of the networks representing the intact pancreatic islets will be presented such as degree distributions, degree correlations and centrality. We shall argue that by analyzing spatial network models of pancreatic islets we can progress our understanding of the interrelation between the structural code and functional connections inside living pancreatic islets.

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# Magnetic domain patterns under an oscillating field

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Magnetic domain patterns show various kinds of structures under a magnetic field. Especially under an oscillating field, one can observe interesting domain patterns such as square/hexagonal lattices, travelling stripe patterns, spirals, concentric circles, etc.

We discuss domain patterns in a ferromagnetic thin film under an oscillating field. The domain patterns consist of up-spin and down-spin clusters. We consider domain patterns with a constant characteristic width. The characteristic width is determined by the balance between the exchange interactions (short-range attractive interactions) and the dipolar interactions (long-range repulsive interactions). We use a simple two-dimensional Ising-like model including those interactions in order to simulate various kinds of domain patterns and discuss the characteristics and mechanism of the patterns.

Under some conditions, lattice patterns are observed in both experiments and numerical simulations. We show numerical simulations of some kinds of lattice patterns. The type of a lattice (i.e. square or hexagonal) depends on the amplitude and frequency of an oscillating field. We also show the numerical simulation of a travelling stripe pattern. In fact, travelling patterns can be observed in experiments. We discuss the mechanism and instability of a travelling pattern. Furthermore, we will also discuss more interesting patterns like spirals and concentric circles.

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# Spatio-temporal modelling of intracellular $Ca^{2+}$ oscillations

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In many non-excitable eukaryotic cells $Ca^{2+}$ oscillations play a key role in intra and intercellular signalling, thus regulating many cellular processes from fertilization to death (Berridge et al., 1998). Mathematical modelling of $Ca^{2+}$ oscillations is of particular importance for understanding the mechanisms underlying these oscillations, and consequently understanding how they may be regulated. The experiments in hepatocytes show that when net plasma membrane $Ca^{2+}$ efflux is reduced, and hence the total concentration of $Ca^{2+}$ in the cell increases, the frequency of $Ca^{2+}$ oscillations increases as well, but importantly, the width of the spikes remains constant (Green et al., 1997, 2002, 2003). The existing mathematical models (for review see Schuster et al., 2002) were not able to explain this phenomenon. We show that these experimental observations can be best explained by taking into account not only the temporal, but also the spatial dynamics underlying the generation of $Ca^{2+}$ oscillations in the cell (Marhl et al., 2008). Here we divide the cell into a grid of elements and treat the $Ca^{2+}$ dynamics as a spatio-temporal phenomenon. Thus converting an existing temporal model into a spatio-temporal one delivers results that are in much better agreement with experimental observations.

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# Wave turbulence in superfluid $^4$He: energy cascades and rogue waves in the laboratory

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A highly excited state of a system with numerous degrees of freedom, characterized by a directional energy flux through frequency scales, is referred to as *turbulent.* Like the familiar manifestations of vortex turbulence in fluids, turbulence can also occur in systems of waves, e.g. turbulence of sound waves in oceanic waveguides, magnetic turbulence in interstellar gases, shock waves in the solar wind and their coupling with Earth's magnetosphere, and phonon turbulence in solids. Following the ideas of Kolmogorov, the universally accepted picture says that nonlinear wave interactions give rise to a cascade of wave energy towards shorter and shorter wavelengths until, eventually, it becomes possible for viscosity to dissipate the energy as heat. Experiments and calculations show that, most of the time, the Kolmogorov picture is correct. We have found, however, that this picture is incomplete. Our experiments with second sound waves in superfluid $^4$He show that, contrary to the conventional wisdom, acoustic wave energy can sometimes flow in the opposite direction.

We shall review briefly the necessary background in turbulence and superfluidity, discussing why superfluid $^4$He is an ideal medium for modelling wave turbulence in the laboratory. Then we will report recent results and discuss their implications in physics and in relation to possible applications such as rogue waves on the ocean.

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# Clustering of inertial particles in turbulent flows

**B. Mehlig**[1)] **and M. Wilkinson**[2)]

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We consider particles suspended in a randomly stirred or turbulent fluid. When effects of the inertia of the particles are significant, an initially uniform scatter of particles can cluster together. We analyse this 'unmixing' effect by calculating the Lyapunov exponents for dense particles suspended in such a random three-dimensional flow, concentrating on the limit where the viscous damping rate is small compared to the inverse correlation time of the random flow (that is, the regime of large Stokes number). In this limit Lyapunov exponents are obtained as a power series in a parameter which is a dimensionless measure of the inertia. We report results for the first seven orders. The perturbation series is divergent, but we obtain accurate results from a Padé-Borel summation. We infer that particles can cluster onto a fractal set and show that its dimension is in satisfactory agreement with previously reported results from simulations of turbulent Navier-Stokes flows [Bec *et al.* 2006].

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# Relative speeds of inertial particles at large Stokes numbers

**B. Mehlig**[1)] **and M. Wilkinson**[2)]

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We discuss the probability distribution of relative speed $\Delta V$ of inertial particles suspended in a highly turbulent gas when the Stokes numbers, a dimensionless measure of their inertia, is large. We identify a mechanism giving rise to the distribution $P(\Delta V) \sim \exp(-C|\Delta V|^{4/3})$ (for some constant $C$). Our conclusions are supported by numerical simulations and the analytical solution of a model equation of motion. The results determine the rate of collisions between suspended particles.

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# Synchronization in large networks of coupled phase oscillators: the effect of network topology

**Edward Ott**

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Synchronization of large systems of coupled oscillators is a basic issue in settings ranging from brain function, to electrical circuits, to laser arrays, etc. This lecture will begin with a review of the paradigmatic Kuramoto model of globally coupled phase oscillators where each oscillator has a different intrinsic frequency drawn from some prescribed probability distribution function [Ott 2002]. We then introduce concepts of network connectivity and formulate the problem of phase oscillators coupled on a network. The problem is analyzed in three stages of approximation. It is found that the effect of network topology on synchronization is mainly characterized by the largest eigenvalue $\lambda$ of the network adjacency matrix. Based on this, one can assess the influence upon synchronizability of such network attributes as diversity in the number of connections to network nodes, spread in the link strengths, and the tendency for highly connected nodes to connect to other highly connected nodes (assortativity).

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# Emergence of collective behavior in large networks of coupled heterogeneous dynamical systems

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After a review and introduction of the subject, this lecture will be devoted to investigations of large systems of coupled heterogeneous dynamical systems. The first part of the lecture will deal with how interactions with external drivers influence the dynamics. Examples in this category include the interaction of the walking dynamics of pedestrians on London's Millennium Bridge that lead to unexpected violent oscillations of the bridge [Strogatz et al 2005, Eckhardt et al 2007] circadian rhythm governing the sleep-awake cycle of animals through the coupling of many oscillatory neurons in the brain's superchiasmatic nucleus influenced by the daily 24 hour variation of sunlight [Antonsen et al], etc. The second part of the talk will be devoted to the study of systems of many coupled heterogeneous dynamical systems, including cases where the coupled systems can be chaotic. As the coupling strength is increased, a transition from incoherent to coherent collective behavior is typically observed. A general theory for this transition will be presented [Ott et al 2002, and Baek and Ott 2004]. An important point is that the macroscopic (i.e., averaged) system behavior is typically periodic even if the systems that are coupled behave chaotically. We present a treatment for the case of global coupling and then indicate how it can be generalized to situations with nontrivial network topology [Restrepo et al 2006].

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# Estimating the state of large spatiotemporally chaotic systems, weather forecasting, etc.

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State estimation is a general requirement for controlling a system or for predicting its future evolution., In this talk we will address the problem of estimating the state of a large spatiotemporally chaotic system from limited noisy measurement data and a a knowledge of a system model. For large systems, state estimation can be particularly challenging because straightforward application of the conventional techniques is typically not feasible due to computational limitations. This problem has very general interest, e.g., for weather forecasting, etc. This talk will present background material, a proposed solution for treating large systems, and illustrative results from application of our technique to weather forecasting and to a laboratory experiment.

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# The Limits of Some Infinite Families of Complex Contracting Mappings

**Dušan Pagon**

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The idea of self-similarity is one of the most fundamental in modern mathematics. The notion of renormalization group, which plays an essential role in quantum field theory, statistical physics and dynamical systems, is related to it. Many fractal and multi-fractal objects, playing an important role in singular geometry, measure theory and holomorphic dynamics, are related. Self-similarity also appears in the theory of $C^*$-algebras (for example in the representation theory of the Cuntz algebras) and in many other branches of mathematics. For the last 25 years the idea of self-similarity demonstrates a growing influence on asymptotic and geometric group theory.

A compact set $F$ in a metric space is called self-similar (in a strong sense) with a similarity coefficient $p$, when it can be divided into $N$ congruent sets, each of which is exactly $p$-times smaller copy of the original set. Our main purpose is a new approach to a broad class of planar fractal sets. We discuss some strongly selfsimilar sets of points in the complex plane, obtained as geometric limits of certain infinite families of contracting mappings (homotheties with $q = \frac{1}{p} < 1$). Some well known fractal sets (like the Sierpinski gasket) are included in this class, but generally new planar fractal sets appear, that have not been studied before.

For an arbitrary stretching factor $r \in (0, 1)$ and a positive angle $\vartheta < \pi$ we define an infinite family of bijective affine mappings of the complex plane, consisting of a rotation of the plane (for a certain multiple of the angle $\vartheta$) with the appropriate stretching (for the related power of factor $r$) and some translation. Starting with a unit segment we repeatedly use this iterated functions system on it. The limiting set of points is under certain conditions in a 1–1 corespondence with the initial unit interval on the real axis.

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# Cyclical interactions, defensive alliances, and the Phoenix effect

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Cyclical interactions are simple yet fascinating and powerful examples of evolutionary processes, able to provide insights into the intriguing mechanisms of Darwinian selection [1] as well as structural complexity [2] and prebiotic evolution [3]. The simplest non-trivial food web describing such cyclical interactions is formed by three species that have relationships analogous to the well-known rock-scissors-paper (RSP) game, where strategies form a closed loop of dominance.

We will first outline some general properties of the RSP game and present Monte Carlo simulations that will help us to get familiar with the basic premise of cyclical interactions. Next, an extension of the basic RSP game, given in the form of a six species model (or food web) will be presented. We will show that the more complex version, supplemented also by heterogeneous invasion rates [4], exhibits many interesting features that can be encountered in the human and animal world. Besides cyclical interactions as the basic ingredient, we will also show and explain the existence of defensive alliances, demonstrate the Phoenix effect, as well as noise-guided evolution [5].

Finally, we will also link the subject with evolutionary game theory [6] and present our recent results showing that static [7] and dynamic [8, 9] stochastic inputs may play a decisive role in determining the evolutionary success of participating strategies. Interested individuals are kindly invited to write me an email⋆ or visit my homepage† to learn more about the subject.

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# Virtual Volatility

**Richard E. Prange, A. Christian Silva**

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A basic investment strategy is: Buy a stock (or portfolio of stocks) at price $S_0$ at time $t = 0$ intending to sell it a 'year' later at time $t = T$ at a higher price $S_T$. A widely adopted semirealistic model for 'predicting' stock prices is the *random walk.* Then the price changes 'daily' by a small fraction $ds_n = (S_{(n+1)dt} - S_{ndt})/S_{ndt} = \mu dt + \sigma\sqrt{dt}W_n$ where $dt = T/N$, $(N >> 1)$ and $W_n$is random with zero mean and unit variance. The Central Limit Theorem then shows that the *predicted* probability distribution function (PDF) of *(log) returns* $R_T = \ln(S_T/S_0)$ is approximately normal (Gaussian). The center of the PDF is the *'expected' return* $\mu_T = N\mu dt$, The PDF's width, $\sigma_T = \sqrt{Ndt}\sigma$. is a version of the *volatility*, commonly but incorrectly equated to the *risk*. Modern Portfolio Theory, MPT,and the Capital Assets Pricing Model CAPM, still in every textbook, make the ridiculous assumption that every market participant knows and indeed agrees on the 'true' values of $\mu_T$ and $\sigma_T$ for each and every portfolio of stocks. Because there is just one exemplar, this PDF *cannot be measured,* even when time $T$ is history. Nevertheless, we agree that the shape is roughly normal. We argue that $\sigma_T$ is known with maybe 20% accuracy and typically is in the range 10%-100%. In contrast, $\mu_T$ is only crudely estimated, e.g. $\mu_T \approx .5\sigma_T \pm \sigma_T$. This triviality implies what is obvious to us, but not to economists, that the actual uncertainty $\sigma_{TV}$of an investor's *predicted return PDF* is *larger* than the random walk (CAPM or Black-Sholes) width, because of inabiity to predict well the expected return parameter $\mu_T$. We call this corrected width the *virtual volatility* VV.

We estimate the size of the effect empirically and find that $\sigma_{TV} \approx 1.3\sigma_T$. We also confirm the normality of the PDF of returns. Remarkably, for a random walk with *fixed* but *unknown* parameters, there is a *bigger* VV effect, $\sigma_{TV} = 1.45\sigma_T$. Our result is evidence of a *market 'anomaly'*, a breakdown of the *efficient market hypothesis,* something not routinely accepted by most academic economists: The simple random walk must be replaced by a *mean reverting random walk.* We extract from the data an empirical *time to revert to the mean of* about 15 months.

The VV also has dramatic consequences on the choice of optimal investment vehicle. While the VV effect obviously reduces the advisability of buying a stock directly, the extra uncertainty actually *improves* the strategy of buying *calls* on the underlying stock. Thus, in this case, *uncertainty* can be a good thing, confirming the adage *ignorance is bliss!*

# Third quantization: a general method to solve master equations for quadratic open Fermi systems

**Tomaž Prosen**

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In this lecture we shall outline a general approach to explicit solution of quantum Liouville equations for open many-body systems out of equilibrium [1].

Using the concept of quantization in the Fock space of operators, the Lindblad master equation for an arbitrary *quadratic* system of $n$ fermions shall be solved explicitly in terms of diagonalization of a $4n \times 4n$ matrix, provided that all Lindblad bath operators are *linear* in the fermionic variables [1].

As an example, the method is applied to the explicit construction of non-equilibrium steady states and the calculation of asymptotic relaxation rates in the far from equilibrium problem of heat and spin transport in a nearest neighbor Heisenberg $XY$ spin 1/2 chain in a transverse magnetic field. Furthermore, we find and demonstrate a novel type of far from equilibrium quantum phase transition with spontaneous emergence of long-range order in spin-spin correlation functions [2].

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# Quantum chaos in many-body systems

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Several approaches to general characterizations of non-integrable interacting many-body systems shall be reviewed [1]. We shall focus on quantum spin 1/2 chains as a particular convenient class of generic systems and outline several dynamical signatures of (non)integrability, employing concepts from random matrix theory (see e.g. [2]) and from quantum information theory (see e.g. [3, 4]). In particular we shall discuss (i) energy level statistics and (generalized) quantum chaos conjecture [5], (ii) decay of dynamical correlation functions with equilibrium relaxation rates as quantum analogues of Ruelle resonances [1], and the relation to quantum transport problem in non-equilibrium statistical mechanics [6], (iii) algorithmic complexity, entanglement production and the efficiency of classical simulations of quantum dynamics [7], and (iv) computation of quantum dynamical entropies [1].

These criteria and their ranges of validity will be discussed and compared, and sometimes quite surprising conclusions are found. Some conjectures and open problems in the ergodic theory of the quantum many problem shall be suggested.

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# Dynamics of Loschmidt echoes and fidelity decay

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Fidelity serves as a benchmark for the reliability in quantum information processes, and has recently attracted much interest as a measure of the susceptibility of general dynamics to external perturbations. A rich variety of regimes for fidelity decay have emerged. In this lecture we shall review some of the most important regimes, and outline the theory that supports them [1]. While we mention several theoretical approaches we use time correlation functions as a backbone for the discussion. Recent experiments in micro-wave cavities and in elastodynamic systems as well as suggestions for experiments in quantum optics shall be discussed.

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# Topology and chaos

**Dušan Repovš**

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In this lecture we shall illustrate, by means of examples, how certain methods of modern geometric topology can be used to construct very interesting and diverse examples in chaos:

(i) Using a classical example from 1930's, due to J. H. C. Whitehead, of an open contractible 3-manifold which fails to be $R^3$, we shall see how the corresponding "continuum at infinity" arises as a chaotic local attractor for a special self-homeomorphism of $R^3$.

(ii) Using interval mappings, we shall demonstrate that every inverse limit space of such mappings can be realized as a global attractor for a homeomorphism of $R^2$.

(iii) Using Borsuk's shape theory, we shall show how one can get a compactum (a certain one-dimensional space called a "solenoid") which cannot be an attractor of any self-map of a topological manifold.

(iv) Using forcing of periodic points in orientation-reversing twist maps of the plane (for example, the Hénon maps), we shall show that an orientation-reversing twist map can be written as the composition of four orientation-preserving twist maps.

(v) Applying the Conley index to the dynamics of $f$, we shall show that period-4 points can be divided into two types and prove that if $f$ is a diffeomorphism, then the existence of a point of type I implies that there exists a compact invariant subset $\Lambda \subset R^2$ such that $f|_\Lambda$ has positive topological entropy.

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# Chaotic motion in rigid body dynamics

**Peter H. Richter**

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The theory of rigid body motion in a gravitational field has a long history. It is intimately connected with the names of Leonard Euler, Joseph Louis de Lagrange, and Sophia Kovalevskaya – each of them identified an integrable family of bodies which now carry their names. The beautiful mathematics of elliptic and hyperelliptic functions was developed in that connection, but little attention was given to the fact that the overwhelming majority of rigid body problems is non-integrable: their typical dynamics is more or less chaotic.

The lecture will outline the nature of the problem and the strategy of its investigation. First we must take into account that realistic rigid bodies need a device to fix a point which is not the center of gravity – a Cardan suspension for example. The configuration space is then no longer SO(3) where Euler's angles or Euler's elegant variables may be used as coordinates, but a 3-torus of Cardan angles. Two other implications are that (i) the set of all these physical systems has (at least) six essential parameters and (ii) the six-dimensional phase space is not always reducible to four dimensions because no component of the angular momentum needs to be conserved. In principle this requires to analyze motion on five-dimensional iso-energy surfaces.

We shall restrict the discussion to cases where at least one component of the angular momentum is conserved so that the motion can be studied on three-dimensional energy surfaces. A first step to get an overview is to identify bifurcation diagrams in the space of conserved quantities (energy and angular momentum), and to determine the topological nature of the various energy surfaces. This is done by looking at the critical points of an effective potential on the reduced configuration space which we call Poisson torus (rather than Poisson sphere). No further analytical work seems to be possible in general; the rest must be done in terms of Poincaré sections.

It is not trivial to define a "good" 2-D surface of section that (i) captures every trajectory on a given energy surface and (ii) may be represented in projection to the Poisson torus. I will report on the state of affairs and present examples.

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# New trends in quantum chaos of generic systems

**Marko Robnik**

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First I shall briefly review the basic elements of the stationary quantum chaos in Hamiltonian systems, the universality classes of energy spectra and eigenfunctions. Then I shall consider the problem of the generic systems whose classical dynamics and the phase portrait is of the mixed type, i.e. regular for certain initial conditions and irregular (chaotic) for other initial conditions. I shall present the so-called Berry-Robnik picture, the Principle of Uniform Semiclassical Condensation (of the Wigner functions of the eigenstates), and the statistical description of the energy spectra in terms of E(k,L) statistics, which is known and shown to be valid in the semiclassical limit of sufficiently small effective Planck constant and is numerically firmly confirmed. Then I shall show the numerical evidence for the deviations from that prediction in mixed type systems at low energies, due to localization and tunneling effects. The most recently developed random matrix model will be shown to apply in this regime, as it is in very good agreement with numerical and experimental data on the so-called mushroom billiards of Bunimovich. Here are also the important open theoretical questions that I shall address.

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# Exact analysis of the adiabatic invariants in time-dependent harmonic oscillator

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The theory of adiabatic invariants has a long history, and very important implications and applications in many different branches of physics, classically and quantally, but is rarely founded on rigorous results. It began with the classical paper by Einstein in 1911, following a suggestion by Lorentz in the same year. We treat the general one-dimensional harmonic (linear) oscillator with time-dependent frequency whose energy is generally not conserved, and analyse the evolution of the energy and its statistical properties, like the distribution function of the final energies evolved from an initial microcanonical ensemble. This distribution function turns out to be universal, i.e. independent of the nature of the frequency as a function of time. The theory is interesting from the mathematical point of view as it comprises elements of the theory of dynamical systems, the probability theory and discrete mathematics, and sheds new light on the understanding of the adiabatic invariants in nonautonomous dynamical systems.

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# Computational algebra and some applications to differential equations

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Consider a system of polynomials

$$f_1(x_1, \ldots, x_n) = 0, \ \ldots, \ f_s(x_1, \ldots, x_n) = 0. \tag{7}$$

The set of all solutions to (1) is called the variety. There are numerical algorithms for solving non-linear systems such as (1). These algorithms solve for one solution at a time, and find an approximation to the solution. They ignore the geometric properties of the solutions space (the variety), and do not take into consideration possible alternate descriptions of the variety (using a different system of polynomials). However recently efficient computational algorithms have been developed which enable us to get algebraic and geometric information about the entire solution space of system (1). They are based on the Gröbner bases theory worked out by B.Buchberger around the middle of 60th of last century.

In the lecture we sketch main ideas of the Gröbner bases theory and discuss some algorithms implemented in computer algebra systems (such as Mathematica, Maple and Singular). We consider applications of Gröbner bases to solutions of systems of polynomial equations and to elimination of variables in such systems. We also present a few applications of the algorithms to the study of some problems of the theory of ordinary differential equations.

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# Self-organized quasiperiodicity in oscillator ensembles with global nonlinear coupling

**Michael Rosenblum** and **Arkady Pikovsky**

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In the classical sense, synchronization of coupled oscillating systems means appearance of certain relations between their phases and frequencies due to weak coupling. After giving brief introduction into the classical theory we review the recent results on self-synchronization in large enesembles of all-to-all interacting units. We discuss experimental examples and the theoretical treatment of the Kuramoto model.

Next, we consider new effects which can appear if the coupling between oscillators is nonlinear in the sense that response of an individual oscillator to a strong driving cannot be taken simply as an "upscaled" response to a weak driving. We formulate a mininal model for am ensemble of nonlinearly coupled oscillators. This model is a generalization of the Kuramoto model. Furthemore, we demonstrate a transition from fully synchronous periodic oscillations to partially synchronous quasiperiodic dynamics. We present an analytic solution of the model that explains the regime where the mean field does not entrain individual oscillators, but has a frequency incommensurate to theirs. The self-organized onset of quasiperiodicity is illustrated with Landau-Stuart oscillators and a Josephson junction array with a nonlinear coupling.

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Rosenblum M and Pikovsky A 2007 *Phys. Rev. Lett.* **98** 054102

# Some Basic Facts about Logical Circuits

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This talk is mainly on the pedagogical side of the conference and hence shall also and in particular address all the students attending. Special effect: The intention is to perform also an electric show-experiment, namely Edisons effect, during the talk.
Let us come to the motivation of the topic: In difference equations and discrete dynamical systems theory, the last ten years stood under an impressive sign: Extending results from autonomous difference equations to non-autonomous ones.
Discrete dynamical systems which have only 0 and 1 as a value, may be interpreted as migrating signals of a logical system in which different operations are possible: These operations are the logical circuit operations AND, OR, NOT.
We review main facts of the elementary operations AND, OR, NOT and see how mathematical modelling of a logical circuit works. We will learn about maxterms, minterms and explore an important representation theorem: There is only one kind of logical operator necessary to describe all finite systems of logical circuits: This is the NOR operator. Alternatively a similar representation theorem holds for NAND operators.
All the circuits considered so far are autonomous ones: The same initial signal constellation at the entries of a system leads always to the same results, independent from the position on the time axis.
In the last part of the talk, will we learn about the non-autonomous scenario of RS-logical circuits which allow to store signals and therefore to store information.
In the whole talk, the corresponding physical realization of a logical operator will be presented: Starting from Edisons effect and ending up with modern semiconductor devices.

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# Transport and localization of Bose-Einstein condensates in disorder

**Peter Schlagheck**

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In my talk, I will present our research activities on transport processes of interacting Bose-Einstein condensates in dimensionally restricted disorder potentials. We study these processes by numerically integrating the Gross-Pitaevskii equation with an external source that simulates the quasi-stationary injection of coherent matter waves onto the disorder region. For the case of one-dimensional disorder potentials, we find that the interaction between the atoms leads to a cross-over from an exponential to an algebraic decrease of the average transmission with the disorder length, which represents a significant deviation from the scenario of Anderson localization. For two-dimensional disorder potentials, the presence of interaction reverts the phenomenon of weak localization and leads to a cone-shaped dip, instead of a peak, in the angle-resolved current of backscattered atoms. Our numerical findings are corroborated by analytical approaches based, in the 1D case, on transfer-matrix-type methods and, in the 2D case, on nonlinear diagrammatic perturbation theory.

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# Solution of Mixed Problem for Elliptic Equation by Monte Carlo and Probability–Difference Methods

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We consider the following problem in domain $\Omega \in R_n$ with border $\partial\Omega$

$$Du(x) \equiv \frac{\partial}{\partial x_i}\left(a_{ij}(x) \cdot \frac{\partial u(x)}{\partial x_j}\right) + b_i(x) \cdot \frac{\partial u(x)}{\partial x_i} + c(x) \cdot u(x) = f(x), \quad \text{in} \quad \Omega, \tag{8}$$

$$\frac{\partial u(x)}{\partial \eta} + d(x) \cdot u(x) = \varphi(x) \quad \text{on} \quad \partial\Omega, \tag{9}$$

where $\frac{\partial u(x)}{\partial \eta} = a_{ij}(x) \cdot \frac{\partial u(x)}{\partial x_j} \cdot cos(\vec{n}, x_i)$. [1].

Coefficients of the operator $D$ and $d(x) \geq 0$ are the limited functions, $f(x) \in H(\Omega)$. Where $H(\Omega) \equiv L^2(\Omega)$. At anyone real $\xi_0, \xi_1, \ldots, \xi_n$ the inequality

$$a_{ij}(x)\xi_i\xi_j - b_i(x)\xi_i\xi_0 - c(x)\xi_0^2 \geq \alpha \sum_{i=1}^{n} \xi_i^2 + \beta\xi_0^2 \tag{10}$$

is realized, $\alpha$ and $\beta$ are positive constants.

The generalized solution $u(x) \in H^1(\Omega)$ of the problem (8), (9) is an element from $H^1(\Omega)$ satisfying integral identity

$$D(u, v) + \int\limits_{\partial\Omega} d(x)u(x)v(x)dx = -(f, v) \tag{11}$$

for $\forall v(x) \in H^1(\Omega)$. If $v = u$ in (11), we get

$$D(u, u) + \int\limits_{\partial\Omega} d(x)u^2(x)dx = -(f, u), \tag{12}$$

by virtue of (10) from which follows

$$\int\limits_{\Omega} \Big(\alpha u_x^2(x) + \beta u^2(x)\Big)dx + \int\limits_{\partial\Omega} d(x)u^2(x)dx \leq -(f, u) \leq ||f||_{H(\Omega)} \cdot ||u||_{H(\Omega)}. \tag{13}$$

A priori estimations for the solution $u(x)$ follows from (13)

$$||u||_{H(\Omega)} \leq \frac{1}{\beta} \cdot ||f||_{H(\Omega)}, \quad ||u_x||^2_{H(\Omega)} \leq \frac{1}{\alpha\beta} \cdot ||f||^2_{H(\Omega)}. \tag{14}$$

Let us consider the case when $\Omega$ is the bounded domain, $u \in C^2(\Omega)$ and operator $D$ given by

$$Du = a_{ij}(x)\frac{\partial^2 u(x)}{\partial x_i \partial x_j} + b_j(x)\frac{\partial u(x)}{\partial x_j} \tag{15}$$

where coefficients $a_{ij}(x)$, $b_j(x)$ and $c(x)$ are real measurable functions defined in $\Omega$.

Let us assume that $a_{ij}(x) = a_{ji}(x)$, that is a matrix of senior coefficients is symmetric.

Let operator $D$ acting in $C^2(\Omega)$ by (15) is elliptic, that is for $\exists\alpha > 0$, $\forall(\xi_1, \ldots, \xi_n) \in R^n$ the inequality $a_{ij}(x)\xi_i\xi_j \geq \alpha|\xi|^2 \equiv \xi_i^2$ is correct.

Let $f$ is measurable functions in $\Omega$. The function $u \in C^2(\Omega)$ is called as the regular solution of the equation $Du(x) = f(x)$, if it satisfies to this equation in each point $x \in \Omega$. Closed domain $\overline{\Omega}$ belongs to the $A^{(k,\lambda)}$ class, if in vicinity of each point $x \in \partial\Omega$ $\partial\Omega$ is given by $\xi_n = \gamma(\xi_1, \ldots, \xi_{n-1})$ in some system of coordinates, $\gamma$ has $k$ continuous derivatives, and also $\gamma^{(k)} \in C^{(0,\lambda)}$, that is it satisfies to Holder condition with index $\lambda$. [2].

For continuous function $\varphi(x)$ on the border $\partial\Omega$, measurable $f(x)$ and elliptic operator $D$ we consider the mixed problem

$$Du(x) = f(x), \quad x \in \Omega, \tag{16}$$

$$\frac{\partial u(x)}{\partial \eta} + d(x)u(x) = \varphi(x), \quad x \in \partial\Omega. \tag{17}$$

Let us assume that $a_{ij} \in C^{(1)}(\Omega)$.

Let $e_i = b_i - \sum\limits_{j=1}^{n} \frac{\partial a_{ij}}{\partial x_j}$, then

$$Du = \frac{\partial}{\partial x_j}\Big(a_{ij}\frac{\partial u}{\partial x_i}\Big) + e_i\frac{\partial u}{\partial x_j} + cu.$$

If $e_i \in C^{(1)}(\Omega)$, then operator $D^*$ acting at function $v(x) \in C^{(2)}(\Omega)$ by equation

$$D^*u = \frac{\partial}{\partial x_j}\Big(a_{ij}\frac{\partial v}{\partial x_i}\Big) + e_i\frac{\partial v}{\partial x_i} + cv$$

called as conjugate to $D$.

Let $\pounds(y,x)$ is Levi function, $T(x) \subset \Omega$, $T(x)$ is a family of domains, for example a family of balls of maximal radius with center in $x$. Let us assume $\pounds(y,x) = 0$ for $y \in \partial T$, then

$$u(x) = \int\limits_{T} \Big(u\, D_y^*\pounds + \pounds f\Big)dy - \int\limits_{\partial T} a_{ij}(y)\,\frac{\partial \pounds}{\partial y_i}\,n_i\,u(y)\,d_yS. \tag{18}$$

The integral equation (18) can be solved by Monte Carlo methods, if the integral operator $K$ of this equation satisfies to condition

$$\|K\|_{H^\infty(\Omega)} = \sup_{x\in\Omega} \int\limits_{T(x)} k(x,y)\,dy < 1, \tag{19}$$

where $k(x,y)$ is a node of the integral equation, it depends on $a_{ij}(x)$, on a derivative $\frac{\partial \pounds}{\partial y_i}$ and on component of an external normal to border of the domain $T$. [2].

At performance of the condition (19) the integral equation (18) can be solved by "random walk by spheres" and "random walk by balls " algorithm of Monte Carlo methods, also it is possible to construct the $\varepsilon$–displaced estimations for $u(x)$. For example, such case is possible, if coefficients of the operator $D$ are constant.

Let $\partial\Omega$ is a surface of Lyapunov, the surface $\Omega$ is convex, coefficients of the operator $D$ are constant. Then the norm of the integral operator acting in $C(\overline{\Omega})$ less than 1. Hence, it is possible to apply Neumann-Ulam scheme to the equation (18).

The integral equation (18) is solved by "random walk by balls" algorithm of Monte Carlo methods. By reaching $\varepsilon$–boundary Markov chain reflected with the probability $p_{\partial\Omega_\varepsilon} = \frac{|a_{ij}|}{|a_{ij}|+|d|}$ and adsorbed with the probability $q_{\partial\Omega_\varepsilon} = \frac{|d|}{|a_{ij}|+|d|}$.

At transition from one condition to the following condition the "weight" of node, that is defined by the recurrence relation

$$Q_0 = 1, \quad Q_{i+1} = Q_i\,\frac{k(x_i, x_{i+1})}{p_\Omega(x_i, x_{i+1})}, \quad i = 0, 1, \ldots,$$

is taken into account. On a border the "weight" of border proportional

$$Q_{\partial\Omega} = \frac{\varphi(x_i)}{|a_{ij}| + |d|}$$

is taken into account.

Let us denote by $h$ a step of the difference scheme in each coordinate direction and by $e_i$ coordinate unit vector in $i$-th coordinate direction. We approximate the domain $\Omega$ and operator $D$ by finite difference method.

Let us define

$$G_h(x) = 2\sum_i a_{ii}(x) - \sum_{i,j,\, i\neq j} |a_{ij}(x)| + h\sum_i |b_i(x)|$$

and assume that

$$a_{ii}(x) - \sum_{j,\, j\neq i} |a_{ij}(x)| > 0, \quad i = 1, 2, \ldots, n.$$

Then we define

$$\Delta t^h(x) = \frac{h^2}{G_h(x)},$$

$$p^h(x, x \pm e_i h) = \frac{a_{ii}(x) - \sum\limits_{j,\, j\neq i} |a_{ij}(x)| + h^2\, b_i^{\pm}(x)}{G_h(x)},$$

$$p^h(x, x + e_i h \pm e_j h) = \frac{a_{ij}^{\pm}(x)}{G_h(x)},$$

$$p^h(x, x - e_i h \pm e_j h) = \frac{a_{ij}^{\pm}(x)}{G_h(x)},$$

$p^h(x, y) = 0$ for the others $x, y \in \Omega \in R_h^n$. Function $p^h(x, y)$ is nonnegative, the sum on $y$ is equal 1 for each $x$. This means that $p^h(x, y)$ is the probability of transitions of some Markov chain that we denote by $\left\{\xi_n^h\right\}$. $p^h(x, y)$ will be coefficients in finite difference approximation. **It is easy to prove that to finite difference mixed problem the Neumann–Ulam scheme is applicable.**

We'll divide a discrete border into the reflecting $\partial\Omega_R^h$ and the absorbing $\partial\Omega_A^h$, then it is possible to construct the $\varepsilon$–displaced approximation of the unique decision in the point $x$. For example, it is possible to construct Markov chain by "random walk by lattices" and to define $\left\{\xi_n^h\right\}$ along this chain. [3], [4], [5], [6], [7], [8].

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# Amphibious complex orbits and its manifestation in quantum mechanics

**Akira Shudo**

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The wavepacket in quantum mechanics has a nonzero transition amplitude to arbitrary regions even when the initial and final regions are separated by energetic or dynamical barriers. Such an aspect has something in common to *classically ergodic systems* in which orbits are allowed to come close to any point in phase space. Apparently, this would be merely an analogy and one may think that these are entirely unrelated to each other since the former is a result of an intrinsic dynamical property but the latter is due to purely quantum effects like tunneling.

However, the studies on complex dynamical systems in more than one dimension have revealed there exists a unique ergodic measure in complex phase space, which has mixing property and thus ergodic (Bedford & Smillie 1991a, 1991b, 1992). A remarkable fact is that it holds not only in hyperbolic systems but also in mixed systems in which quasiperiodic or chaotic orbits coexist in phase space.

These mathematical results predict that the transition between classically disconnected regions occurs under the same mechanism as in classically ergodic systems, and that the orbits connecting classically disjointed components should have *amphibious* nature (Shudo *et al* 2008a, 2008b).

Here, we show that amphibious complex trajectories explain the existence of *amphibious states* which were found as the quantum states ignoring underlying classical invariant structures (Hufnagel *et al* 2002, Bäcker *et al* 2005). We also present the results of numerical investigations for a coupled kicked oscillator, in which the amphibious states are realized without a special tuning of the system (Ishikawa *et al* 2007, 2008).

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# Toward pruning front theory for the Stokes geometry

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The saddle point method is known to be an efficient technique to approximately evaluate integrals. A difficulty in applying the saddle point method in general is that not all of the saddles point solutions necessarily contribute. The *Stokes geometry* carries all geometrical information to judge which saddles should appear in its evaluations and which should be dropped, thereby it tells us how to construct global asymptotic solutions from the local pieces.

Here we present an idea for developing bifurcation theory of the Stokes geometry in quantum maps whose classical counterpart exhibit chaos (Shudo 2007, Shudo and Ikeda 2008). A concrete recipe to give the complete Stokes geometry is presented for the quantum Hénon map, in which some new ingredients absent in conventional theory of asymptotics (Aoki et al 1994, 2005a, 2005b, Howls 2004) are taken into account.

A key strategy is to first establish the Stokes geometry in the horseshoe limist, in which the underlying classical dynamics is described by the binary symbolic dynamics and the corresponding Stokes geometry is also analytically tractable, and then to trace it as a function of the system parameter by focusing on bifurcation of the Stokes geometry. The approach exactly follows the *pruning front theory* of classical symbolic dynamics (Cvitanović et al1988).

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# Coupled oscillators: Why may they be used to describe cardiovascular dynamics?

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For a healthy human in repose the heart expels blood about once per second in an oscillatory manner. An amount of blood equivalent to the total amount (5.0–5.5 l) is circulated in about one minute. During this interval, which may be considered as an average circulation time for blood around the human body, five oscillatory components have been observed. They correspond to control mechanisms that reduce the velocity of flow, enabling the cells that surround the capillary bed to exchange energy and matter efficiently with the blood, thereby realizing the ultimate goal of the circulation. At the moment, this story, can be reconstructed only partially, because –

(i) The notion of flow is meaningful only at the macroscopic level, where many processes coexist and interact, whereas microscopic studies of the mechanisms of vasomotion are of limited use.
(ii) Non-invasive techniques to record blood circulation in small vessels (based on laser Doppler measurements) facilitate recordings of the skin flow. At greater depths, only the flow in the larger vessels can be assessed (by ultrasound techniques).
(iii) Currently available methods of time series analysis allowing for reconstruction of time-variable complex dynamics are limited in efficacy.

The development of data analysis methods for complex oscillatory dynamics following Grassberger and Proccacia's (1983) algorithm for calculation of the correlation dimension will be reviewed, and the advantages and pitfalls of the currently used methods will be discussed. In addition, the possibilities of the phase dynamics approach for reconstructing the dynamics properties of living systems will be considered, as will also the importance of the uncertainty principle. Methods for the detection of synchronization and direction of coupling, and the importance of surrogate data in hypothesis testing, will be outlined. The current state-of-the art understanding of the cardiovascular system as a system of coupled oscillators will be reviewed.

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# Can the brain and the cardiovascular system talk to each other and, if so, how?

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The role of the central nervous system in controlling the cardiovascular system has been discussed in numerous studies. The basic functional unit of the nervous system – the action potential – has been understood in great detail. In their pivotal experiments, however, Hodgkin, Huxley and Katz (1952) clamped the membrane potential at a constant value so that, ever since, its temporal dynamics was neither investigated nor properly understood. *In vivo*, the ionic concentrations fluctuate continuously as part of the normal function of the cardiovascular system. Recently developed non-invasive imaging techniques allow for understanding of the role of glial cells in mediating the ionic concentrations of the neurones in the brain (Schipke *et all*, 2008), thereby opening up the question of dynamical interactions between the cardiovascular oscillations and the brain waves.

We will review recent studies based on the phase dynamics approach that have began to illuminate possible bidirectional causal interactions between the two systems. We show that the $\delta$ waves extracted from an EEG signal, and the phase of respiration, are coupled differently during deep and light anæsthesia (Musizza *et all*, 2007). Moreover, using fMRI, a characteristic frequency of 0.03-0.04 Hz was demonstrated in the brain (Horovitz *et all*, 2008). This frequency was already known to exist in the cardiovascular system. The characteristic frequencies of these two major systems of the human organisms – the cardiovascular that takes care of energy and matter exchange, and the neuronal system that takes care of information transfer within the body – will be summarized, pointing to the new opportunities opened up by the phase dynamics approach for reaching an understanding of how the two system interact so efficiently.

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# Microwave studies of chaotic systems

## Lecture 1: Currents and vortices

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In flat quasi-two-dimensional microwave resonators there is a one-to-one correspondence between the Helmholtz equation for the electric field pointing from the bottom to the top, and the stationary Schrödinger equation for the corresponding quantum-mechanical billiard system. This offers the unique possibility to test theories, originally developed for quantum-mechanical systems, in their microwave analogue (Stöckmann 1999). I shall illustrate these features in a series of three lectures.

In closed systems the wave functions are characterized by meandering patterns of nodal lines and domains, the universal features of which can be well described by percolation theory. In open systems there are no longer nodal lines, but only nodal points, corresponding to vortices of the quantum-mechanical flow. Here the random plane wave approach has proven to be a powerful method to describe the universal features of the wave fields. The model allows for an easy calculation of various distribution functions of currents, vortex densities etc., as well as various pair correlation functions. In microwave experiments a large number of predictions from theory could be verified (Kuhl et al. 2007).

In systems with a potential landscape the model is no longer applicable. Here the formation of caustics is possible, leading to branch-like structures in the flow as observed on scanning tunneling microscopy (STM) experiments on quantum point contact structures (Topinka et al. 2001). In microwave billiards potentials may be realized by introducing appropriately shaped scatterers into the resonator. By this the STM results could be qualitatively confirmed. The very same model has been also used for the formation of wave patters in the sea due to locally varying velocity fields (Heller et al. 2008), thus allowing at the same time for the study of non-Gaussian wave patterns (rogue waves etc.) in the ocean.

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# Microwave studies of chaotic systems

## Lecture 2: Line width distributions in open systems


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In a typical microwave experiment from the transmission between or refection at various antennas the system properties are to be determined. There is the obvious question how to distinguish between the system properties and those of the connecting cables, antenna connectors etc.. Scattering theory, originally developed in nuclear physics, is the method of choice to cope with this situation (Guhr et al. 1998). The scattering matrix is given by

$$S(E) = 1 - 2\imath W^\dagger \frac{1}{E - H_{\text{eff}}} W \tag{20}$$

where $H_{\text{eff}} = H - \imath W W^\dagger$. The diagonal elements $S_{ii}$ of the scattering matrix are the reflection amplitudes at the $i$th antenna, whereas the non-diagonal elements $S_{ij}$ are the transmission amplitudes between antenna $i$ and $j$. $H$ is the Hamiltonian of the unperturbed system, and the $W$ matrix contains the information of the coupling of the $i$th antenna to the $n$th wave function. There is a vast amount of theoretical studies (Beenakker 1998), as well as a number of microwave results (Kuhl et al. 2005). Up to now, however, apart from one exception (Persson et al. 2000), only average quantities such as the distribution of reflection or transmission coefficients have been studied. The poles of the scattering matrix had not been accessible, however, in the regime of strong overlap. Here recently a breakthrough has been achieved. By an application of the method of harmonic inversion (Wiersig, Main 2008) the poles of the scattering matrix could be resolved in a regime, were the line widths exceed the mean level spacings by a factor of 10 and even more. By this it became possible to test theories on the distribution of line widths in chaotic systems (Sommers et al. 1998), which seemed to be unaccessible experimentally up to now.

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# Microwave studies of chaotic systems

## Lecture 3: A random matrix approach to fidelity


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The concept of fidelity has been introduced more then 20 years ago by Peres (1984) as a quantitative measure to characterize the quantum-mechanical stability of a system against perturbations. The fidelity amplitude is defined as the overlap integral of a wave packet $|\psi_0\rangle$ with itself, after the evolution under two slightly different Hamiltonians,

$$f(t) = \left\langle \psi_0 \left| e^{\frac{\imath}{\hbar}(H+\lambda V)t} e^{-\frac{\imath}{\hbar}Ht} \right| \psi_0 \right\rangle \tag{21}$$

where $\lambda V$ is a small perturbation of $H$. There had been a renewed interest in fidelity because of its obvious relevance for quantum computing.

Most perturbations studied up to now had been global, which can be treated theoretically by taking $H$ and $V$ from one of the Gaussian ensembles. In billiards a global perturbation can be achieved, e. g., by the shift of one wall. Local perturbation had been overlooked, which is surprising, since most, if not all, perturbations in real systems are of the local type, such as a diffusive jump of an atom, or a spin-flip. Only recently the fidelity decay due to a local perturbation has been studied as well, realized in a microwave billiard by the shift of a small scatterer (Höhmann et al. 2008). For local perturbations it is easy to calculate the fidelity decay analytically, which shows up to be algebraically, again by using the random plane wave approach.

For global perturbations supersymmetry techniques can be used to calculate the Gaussian averages analytically (Stöckmann, Schäfer 2005). In this case the long-time decay is either Gaussian or exponential. On this occasion a short tutorial introduction into the concept of supersymmetry will be given. We shall see that the underlying ideas are simple, and often exact results are easy to obtained. Unfortunately mostly the resulting integrals are very complicated, and much additional effort is needed to bring the solution, though exact, into a form which is suitable for practical purposes.

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# On the mechanism of quantization of chaos.
# - Phase quantization -

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Since the early stage of the study of Hamilton chaos, semiclassical quantization based on the low-order Wentzel-Kramers-Brillouin (WKB) theory, the primitive semiclassical approximation to the Feynman path integrals (or the so-called Van Vleck propagator), and their variants have been suffering from difficulties such as divergence in the correlation function, and so on. Even the celebrated Gutzwiller trace formula is not an exception. It is widely recognized that the essential drawback of these semiclassical theories commonly originates from the erroneous feature of the amplitude factors in their applications to classically chaotic systems. This makes a clear contrast to the success of the Einstein-Brillouin-Keller quantization condition for regular (integrable) systems. We show here that energy quantization of chaos in semiclassical regime (characterized with a small Planck constant) is, in principle, possible in terms of constructive and destructive interference of phases alone, and the role of the semiclassical amplitude factor is indeed negligibly small, as long as it is not highly oscillatory [1]. To do so, we first sketch the mechanism of semiclassical quantization of energy spectrum with the Fourier analysis of phase interference in a time correlation function, from which the amplitude factor is practically factored out due to its slowly varying nature [2]. In this argument there is no distinction between integrability and nonintegrability of classical dynamics. Then we present numerical evidences that chaos can be indeed quantized by means of amplitude-free quasi-correlation functions. This is called phase quantization [1]. Finally, we show explicitly that the semiclassical spectrum is quite insensitive to smooth modification (rescaling) of the amplitude factor [3]. At the same time, we note that the phase quantization naturally breaks down when the oscillatory nature of the amplitude factor is comparable to that of the phases, which is quite likely to be materialized in a very high energy case and/or in dynamics on a hard potential function like the stadium billiard. Such a case generally appears when the Planck constant of a large magnitude pushes the dynamics out of the semiclassical regime. We will show a possible route to such a regime beyond the standard WKB theory.

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# Quantum wavepacket bifurcation and entanglement in molecules. - Generalization of classical mechanics -

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I discuss characteristic dynamics of a system consisting of classical and quantum subsystems strongly coupling with each other. Typically, the quantum subsystem is composed of light and fast particles, while heavy and slow particles constitute the classical subsystem. Such systems are quite ubiquitous, since molecules are more or less approximated in terms of this mixed quantum-classical representation, with nuclei being heavy and classical particles and electrons being light and quantum particles. More explicitly, the separation of electronic and nuclear motions based on this idea constitutes the heart of the so-called Born-Oppenheimer approximation, which is now the standard concept in material sciences. In the Born-Oppenheimer view, nuclei are supposed to propagate in time (either classically or quantum mechanically) on a potential energy function, which is given as electronic energy at parametrically fixed nuclear coordinates.

We first examine the validity range of the Born-Oppenheimer (BO) approximation with respect to the variation of the mass ($m$) of negatively charged particle by substituting an electron ($e$) with muon ($\mu$) and antiproton ($\bar{p}$) in hydrogen molecule cation [1]. With use of semiclassical quantization applied to ($ppe$), ($pp\mu$), and ($pp\bar{p}$) under a constrained geometry, we estimate the energy deviation of the non-BO vibronic ground state from the BO counterpart. It is found that the error in the BO approximation scales to the power of 3/2 to the mass of negative particles, that is, $m^{1.5}$. The origin of this clear-cut relation is analyzed based on the original perturbation theory due to Born and Oppenheimer, and it is shown the first correction to the BO approximation should arise from the sixth order term that is proportional to $m^{6/4}$.

We next proceed to the breakdown of the Born-Oppenheimer approximation, in which two (or more) adiabatic potential energy functions undergo near degeneracy (the so-called avoided crossing). In the vicinity of this avoided crossing, the electronic and nuclear motions strongly couple and thereby entangle. In passing this region, both the electronic and nuclear wavepackets bifurcate in the standard manner of quantum entanglement. We show such wavepacket bifurcation can be directly observed in terms of the femtosecond pump-probe photoelectron spectroscopy. As an illustrative example, we show a (theoretically mapped) movie of such a wavepacket bifurcation arising from the nonadiabatic (non-Born-Oppenheimer) motion on the ionic ($Na^+I^-$) and covalent (NaI) states of sodium iodide [2].

Classical trajectory study of nuclear motion on the Born-Oppenheimer potential energy surfaces is now one of the standard methods of molecular dynamics (as in the simulation of protein dynamics). However, as soon as more than a single potential energy surface are involved due to nonadiabatic coupling as above, such a naive application of classical mechanics loses its theoretical foundation. This is a classic and fundamental issue in the foundation of molecular and material science. To cope with this problem, we propose a generalization of classical mechanics, in which the force appears as a matrix with the dimension equal to the number of the adiabatic states involved. With this matrix force, the "classical" paths passing across the avoided crossing region undergo branching associated with the simultaneous electronic wavepacket bifurcation, thereby making the quantum-classical description of entanglement possible [3]. Numerical examples along with relevant graphics will be presented in a realistic molecular system.

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# Model based development of desynchronizing brain stimulation techniques I: Reshaping neural networks

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Within the last years standard high-frequency (HF) deep brain stimulation became the standard therapy for medically refractory movement disorders. HF deep brain stimulation has been developed empirically, mainly based on observations during neurosurgical procedures. In contrast, to overcome limitations of standard HF deep brain stimulation, we use a model based approach. We make mathematical models of affected neuronal target populations and use methods from statistical physics, nonlinear dynamics, and synergetics to develop mild and efficient control techniques (Tass 1999; 2003; Hauptmann et al. 2007a, 2007b; Tass et al. 2008; Popovych et al. 2005; Omel'chenko et al. 2008). We specifically utilize dynamical self-organization principles and plasticity rules. In this way, we have developed multi-site coordinated reset (MCR) stimulation, an effectively desynchronizing brain stimulation technique (Tass 2003). The goal is not only to counteract pathological synchronization on a fast time scale, but also to unlearn pathological synchrony by therapeutically reshaping neural networks (Tass & Majtanik 2003). We examined the effects of MCR stimulation in 20 patients with severe Parkinson's disease or essential tremor during the first week after electrode implantation with a novel portable brain stimulator. According to our theoretical predictions, in all 20 patients epochs of MCR stimulation caused pronounced therapeutic effects, which outlasted MCR stimulation during the whole post-MCR observation period prior to dismissal (i.e. during at least four days).

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# Model based development of desynchronizing brain stimulation techniques II: Target point diagnosis and restoring physiological connectivity

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High-frequency test stimulation for tremor suppression is a standard procedure for functional target localization during deep brain stimulation. This method does not work in cases where tremor vanishes intraoperatively, e.g. due to global anesthesia or due to an insertional effect. To overcome this difficulty, we developed a stimulation technique that effectively evokes tremor in a well-defined and quantifiable manner. For this, we used patterned low-frequency stimulation (PLFS), i.e. brief high-frequency pulse trains administered at pulse rates similar to the neurons' preferred burst frequency (Barnikol et al. 2008). In a computational investigation of an oscillatory neuronal network temporarily rendered inactive, we found that already at weak stimulus intensities PLFS evokes synchronized activity, phase locked to the stimulus. We applied PLFS to a patient with spinocerebellar ataxia type 2 (SCA 2) with pronounced tremor that disappeared intraoperatively under general anesthesia. In accordance with our computational results, PLFS evoked tremor, phase locked to the stimulus (Barnikol et al. 2008). In particular, weak PLFS caused low-amplitude, but strongly phase-locked tremor (Barnikol et al. 2008). PLFS test stimulations provided the only functional information about target localization. Optimal target point selection was confirmed by excellent postoperative tremor suppression (Freund et al. 2007; Barnikol et al. 2008). The clinical results obtained in our SCA 2 patient with permanent high-frequency stimulation are in accordance with a computational study (Hauptmann & Tass 2007). Additionally taking into account physiological input into neuronal populations, we revealed that multi-site coordinated reset stimulation is able to restore physiological connectivity (in preparation). Implications thereof will be explained in detail.

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# Dynamical Reaction Theory

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We will discuss the dynamical reaction theory for multi-dimensional Hamiltonian systems. Roughly speaking, the dynamical processes of reactions consist of the following three, i.e., redistribution of energy among vibrational modes in the potential well, going over the potential saddles, and their dynamical connection. For each of the former two processes, perturbation approach is possible to construct the normal form. For the distribution, the normal form describes how the vibrational modes exchange energy in the well. When nonlinear resonance takes place, the perturbation theory breaks down, and energy exchange among vibrational modes is enhanced dramatically there. In the action space, resonant regions constitute the network called the Arnold web. Therefore, properties of the Arnold web play an important role in our topics. For the processes of going over the saddle, the normal form theory is developed recently, which provides a mathematically sound foundation of the concept of transition states (TSs). The theory is based on the phase space structures called normally hyperbolic invariant manifolds (NHIMs). It enables us to define the boundary between the reactant and the product, and to single out the reaction coordinate near the saddles of index one.

After reviewing these concepts, we discuss the following three subjects. First, we present existence of fractional behavior for processes in nonuniform Arnold webs. We also discuss reaction processes under laser fields utilizing cooperative effects of laser fields and the Arnold web. This offers a possibility of manipulating reaction processes by designing laser fields. Second, We discuss that the conventional idea of reaction coordinates is not valid when the condition of normal hyperbolicity is broken. These situations take place when the values of tangential Lyapunov exponents on the NHIMs become comparable to those of the normal exponents. Third, we discuss the network of intersections between stable/unstable manifolds emanating from the NHIMs in multi-dimensional Hamiltonian systems. We point out that the network enables the chaotic itinerancy in Hamiltonian dynamical systems.

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# Granular Matter Dynamics

# I. The Many Phases of Vibrated Granular Matter

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Vibrated granular matter can behave in an amazing variety of ways, ranging from solid-like when the shaking is weak, to fluid-like and even gas-like behavior at strong shaking [Jaeger *et al.* 1996; Aranson and Tsimring 2006]. In this lecture we bring some order in this dynamical labyrinth by constructing a *phase diagram* for vertically shaken granular matter in a 2D container [Eshuis *et al.* 2007].

At weak shaking, the observed phenomena (a bouncing bed, which starts to exhibit standing waves like a violin string when the shaking is increased) are well described by mechanical models [Sano 2005; Eshuis *et al.* 2007].

At stronger shaking, the phenomena are better described by granular hydrodynamics [Goldhirsch 2003]. For increasing shaking strength, the standing waves first give way to the Leidenfrost state (in which a dense cloud of particles is held afloat by a gas-like layer of fast particles underneath [Eshuis *et al.* 2005]), followed by a transition to buoyancy driven convection: The particles now form counter-rotating rolls very similar to Rayleigh-Bénard convection rolls in an ordinary fluid [Khain and Meerson 2003; Eshuis *et al.* 2008]. Finally, at even higher shaking strengths the grains are swept into a gaseous state, moving wildly throughout the container. We will focus especially on the hydrodynamic description of the phenomena at strong shaking.

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# Granular Matter Dynamics

## II. Faraday's heaping effect unravelled

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One of the classic and most enigmatic phenomena exhibited by granular matter is Faraday heaping: When a bed of fine sand is vertically vibrated, its initially flat surface breaks into a landscape of heaps, which in the course of time tend to coarsen into a single larger heap [Faraday 1831; Behringer *et al.* 2002]. As was already suggested by Faraday, the *surrounding air* must play a crucial role in this process since at very low pressures, when the air drag on the particles can be ignored, no heaping of any kind is observed [Pak *et al.* 1995].

Until recently, the heaping had been studied mostly experimentally, giving rise to several rivaling explanations: internal avalanches [Laroche *et al.* 1989], horizontal pressure gradients [Thomas and Squires 1998], and enhanced stability of inclined surfaces [Duran 2002]. Here we describe numerical simulations, closely corresponding to experiment, that confirm the horizontal pressure mechanism of Thomas and Squires. We show that the heaps are formed by the air that flows - through the bed - into the void beneath the bed when this detaches from the vibrating bottom. Our simulations also explain the eventual coarsening into one single heap [Van Gerner *et al.* 2007].

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# Chaotic scattering in the regime of weakly overlapping resonances

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Chaotic quantum scattering occurs when Schrödinger waves are scattered by a system with chaotic classical dynamics. The eigenvalues of the system manifest themselves as resonances with average spacing $D$ and average width $\Gamma$. For chaotic systems, the spectral fluctuation properties of these eigenvalues coincide with the predictions of random–matrix theory (RMT) for matrices of the same symmetry class ("Bohigas–Giannoni–Schmit conjecture"). For time–reversal invariant systems, the matrix ensemble is the Gaussian Orthogonal Ensemble (GOE) of real and symmetric matrices.

The theory of chaotic scattering uses RMT or related models to predict average cross sections and correlation functions of scattering amplitudes in all three regimes, the regime of isolated resonances ($\Gamma \ll D$), the regime of weakly overlapping resonances ($\Gamma \approx D$), and the regime of strongly overlapping resonances ($\Gamma \gg D$) (the "Ericson regime"). After reviewing some results for $\Gamma \ll D$ and for $\Gamma \gg D$ (where the theory has been thoroughly tested), I will focus attention on the comparison of recent experimental results on microwave billiards of the Darmstadt group with theory in the regime $\Gamma \approx D$.

# Preponderance of ground states with spin zero and/or positive parity

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To explore generic features of the nuclear shell model, one often uses a random–matrix model, the two–body random ensemble (TBRE). In the TBRE, the two–body matrix elements of the shell model are taken to be Gaussian–distributed random variables with mean values zero and a common second moment. The single–particle states within a major shell are assumed to be degenerate. In 1998, Johnson, Bertsch, and Dean observed that in the TBRE, ground states with spin zero occur much more frequently than corresponds to their statistical weight. In 2004, Zhao *et al.* showed that in the TBRE, states with positive parity are likewise favoured ground states. That "predominance" of ground states with fixed quantum numbers occurs also in atoms and for bosons and has, therefore, found wide attention.

In the lecture, I will present these surprising facts and the explanations that have been put forward in the literature. I will then focus attention on a model which permits a detailed analytical and numerical study of the preponderance of ground states with positive parity. The model, worked out in collaboration with Thomas Papenbrock, leads to a thorough understanding of the phenomenon.

# Spectral Fluctuation Properties of Constrained Ensembles of Random Matrices

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Wigner's random matrices enjoy wide applications in many fields of physics. Depending on the symmetries of the system, one uses one of Dyson's three canonical random–matrix ensembles to model successfully spectral fluctuation properties of or chaotic scattering on the system. That success of canonical random–matrix theory (RMT) is somewhat surprising since typically, the structure of the Hamiltonian of the system differs from that of a Gaussian random matrix: It may, for instance, be sparse or banded.

More realistic random–matrix models that take into account such structural details lack the invariance properties of the canonical ensembles. This is why it has so far not been possible to derive the spectral fluctuation properties of these ensembles analytically. A recent approach to such ensembles views them as constrained ensembles (some matrix elements of the canonical ensembles are constrained to vanish).

In the lecture, I will introduce the three canonical ensembles of RMT. I will then motivate the use of constrained ensembles, and I will present analytical results on level repulsion and on spectral fluctuation properties that have recently been obtained in collaboration with Thomas Papenbrock and Zdenek Pluhar.

# Planet formation

**Michael Wilkinson**[1], **Bernhard Mehlig**[2]

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### Lecture 1: Observational evidence and review of theories

The theoretical basis for understanding the formation of the planets in our own solar system is far from secure. Also, in the last decade several hundred planets have been discovered orbiting other stars (Butler *et al* 2006) and these discoveries have included some surprises. For example, there are numerous extra-solar planets with very eccentric orbits, and gas-giant planets which orbit close to their star.

The standard theories for planet formation (Safranov, 1969, Goldreich and Ward 1973) involve building up planets from dust grains which are suspended in the gas of the circumstellar nebula surrounding young stars. I will review the standard models for the growth of planets starting from the aggregation of dust particles, and describe some of the difficulties faced by these models. I shall describe some recent attempts to circumvent these difficulties by embellishments of the standard models (Johansen *et al*, 2007, Kretke and Lin, 2007). At the end of the lecture I will mention a new and radically different theory which we have proposed (Wilkinson and Mehlig, 2008).

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**Lecture 2: Collision rates and relative velocities of turbulent aerosols**

The standard model for planet formation involves the aggregation of dust particles in a circumstellar accretion disc. It is surmised that the gas in the accretion disc must be turbulent in order to explain the relatively short lifetime of accretion disc. This makes it necessary to understand the dynamics of dust particles in a turbulent gas. I will review some basic properties of turbulent flows (Frisch, 1995), and describe some recent results on the relative velocities and collision rates of the particles (Wilkinson *et al*, 2006, Mehlig *et al*, 2007, Gustavsson *et al*, 2008).

I shall also describe how the properties of the gas in the circumstellar accretion disc can be estimated from a steady-state theory with very few input parameters, using a theory of Shakura and Sunyaev (1973).

The dust particles are bound by van der Waals and other weak electrostatic forces. I will describe some estimates for the collision speed at which aggregates of dust particles will be fragmented (Dominik and Tielens, 1997), and show that the relative speeds of colliding dust aggregates make them very vulnerable to being fragmented (Wilkinson *et al*, 2008). This is a strong indication that an alternative theory must be sought.

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**Lecture 3: Can gravitational direct collapse create planets?**

I shall describe an alternative hypothesis for the formation of planets, developed in collaboration with Bernhard Mehlig, which we term *Concurrent Collapse*. According to our hypothesis, when a cloud of interstellar gas collapses to form a star, it fragments, giving rise to smaller objects which are gravitationally bound to the star. These *juvenile planets* are initially formed in non-circular orbits and have an elemental composition which is representative of the star. The juvenile planets can interact with the accretion disc in such a way that their orbit and their composition can be dramatically changed. Collisions between juvenile planets are also possible. Our hypothesis avoids having to resolve the difficulties faced by the dust aggregation model. It also provides satisfying explanations for the existence of exoplanets with eccentric orbits, for the occurrence of FU Orionis outbursts and for the melting of *chondrules* found in meteorites. Our explanation will be contrasted with those offered in the framework of the standard models (Ford *et al* 2003, Lodato and Clark, 2004, Hewins, 1997).

The concurrent collapse hypothesis depends upon the assumption that a gravitationally collapsing gas cloud will fragment. Gravitational collapse is imperfectly understood process. I will describe Jeans' theory for determining the size of objects formed by gravitational collapse, and discuss some of the arguments which have been advanced in the past to support the view that collapse is normally accompanied by fragmentation (Hoyle 1953, Low and Lynden-Bell, 1976, Padoan and Nordlund, 2002). None of these are really satisfactory. I will present an outline for a new theoretical approach to this problem.

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# Bifurcations, order, and chaos in Bose-Einstein condensates with long-range interactions

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Bose-Einstein condensates with long-range atomic interactions, in addition to the contact interaction, open the possibility of studying the properties of degenerate quantum gases in which the relative strengths of long- and short-range interactions can be continuously adjusted by tuning the contact interaction via a Feshbach resonance. Examples are Bose-Einstein condensates of neutral atoms with electromagnetically induced attractive 1/r interaction (O' Dell et al. 2000) and dipolar Bose-Einstein condensates (Santos et al. 2000). The achievement of Bose-Einstein condensation in a gas of chromium atoms (Griesmaier et al. 2005), with a large dipole moment, has opened the way to promising experiments on quantum gases with long-range interactions (Koch et al. 2008). In both types of condensates universal stability thresholds exist where collapse of the condensates sets in. We show that these thresholds in fact correspond to bifurcation points where always two solutions of the Gross-Pitaevskii equation disappear in a tangent bifurcation (Papadopoulos et al 2007), one dynamically stable and the other unstable. We point out that the thresholds also correspond to "exceptional points" (Cartarius et al 2008a, b), i.e. branching singularities of the Hamiltonian. We analyze the dynamics of excited condensate wave functions via Poincaré surfaces of section and find both regular and chaotic motion, corresponding to (quasi-) periodically oscillating and irregularly fluctuating condensates, respectively (Wagner et al 2008). Stable islands are found to persist up to energies well above the saddle point of the mean field energy, alongside with collapsing modes.

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# Cold atoms in optical lattices with disorder

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Cold atoms in optical lattices form a wonderful toolbox (Jaksch and Zoller 2005) for creating novel matter phases realizing a "condensed matter theorist dream". This comes from an unprecedented control of the parameters in cold atoms physics where both the depth of the optical lattice (changing laser intensity or detuning) as well as atom-atom interactions (via external magnetic field and Feshbach resonances) can be modified with great precision. This allows for observation of quantum phase transitions (Sachdev 2001) as exemplified for the Bose-Hubbard (BH) tight binding model (Fisher et al. 1989, Jaksch et al. 1998, Greiner et al. 2002). Novel phases can be developed by studying Fermi-Bose or Bose-Bose mixtures as well as for the so called spinor condensates (where atoms in a few Zeeman sublevels are trapped together in optical traps) - review of Lewenstein et al. (2007) is a valuable introduction to that subject.

For a quantum chaos community particularly interesting may be studies of disordered systems. Here cold atomic settings allow to introduce disorder n a controllable and repeatable way using optical potentials created by laser speckles or bichromatic lattices (Damski et al.2003, Roth and Burnett 2003). This opened up exciting possibilities for a clear observation of Anderson localization for weakly interacting bosons. This effort, after preliminary, discouraging results (Clément et al. 2005, Fort et al. 2005, Schulte et al. 2005) has led to a spectacular success quite recently (Billy et al. 2008, Roati et al 2008). For strongly interacting bosons in a deep optical lattice the existence of a novel phase called Bose-glass phase has been predicted (Giamarchi and Schulz 1988, Fisher et al. 1989). Recently, the first attempt to produce this phase with ultracold atoms in a bichromatic quasi-disordered optical lattice has been reported (Fallani et al.07). This experiment is analysed using Time-evolving block-decimation (TEBD) algorithm of Vidal (2003).

The work supported by Marie Curie ToK project COCOS (MTKD-CT-2004-517186).

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# Sources of ultra-high energy cosmic rays

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One of the most fascinating puzzles in particle astrophysics today is that of the origin and nature of the highest energy cosmic rays. These particles have energies orders of magnitude beyond even the future capabilities of any earthly particle accelerator. Such energies are so extreme that they could arise in only the most violent places in the universe.

The Pierre Auger Observatory is a major international effort to make precise, high statistics studies of the cosmic rays with energies above $10^{19}$ eV. The Southern Observatory was constructed in Province of Mendoza, Argentina and is designed to work in a hybrid mode incorporating both a ground array of 1.600 particle detectors spread over 3.000 $km^2$ with fluorescence telescopes placed on the boundary of the surface array.

First data collected by the P. Auger Observatory provide evidence for anisotropy in the arrival directions of the cosmic rays with highest energies, which are correlated with the positions of relatively nearby Active Galactic Nuclei.

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# Entanglement and random quantum states

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Quantum entanglement is one of the properties that makes quantum theory "strange" and different from classical physics. While the concept of entanglement has been known from the very beginning of quantum theory, only relatively recently have experiments advanced to the point of enabling manipulation of individual quanta. What is more, it has been realized that the entanglement can be used to our advantage. It enables for powerful quantum protocols that are better than the best classical procedures. In the lecture we shall address two questions: (i) what is the entanglement of typical quantum states, and (ii) how it comes that there is apparently no entanglement in macroscopic world. To explain the latter we will use properties of typical quantum states, that is states drawn according to unitarily invariant Haar measure.

In the first part we shall quantify the entanglement of random quantum states by calculating the average values of Schmidt coefficients. We will also study protocols for generating random quantum states. Efficiency of the protocol can be expressed in terms of a gap of Markovian process. In the second part we will show how to use properties of random states to explain the lack of entanglement for macroscopic systems. There will be two issues, practicality of detection and the role played by generic initial conditions. First, we shall show that the detection of entanglement in random quantum states is very hard, in fact, it gets exponentially hard with the number of particles. Second, we shall show that if the initial condition is a generic separable state then time evolution with an arbitrary hamiltonian will after very short time result in a state having no entanglement between subsystems of few degrees of freedom.

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# Random Matrices, Quantum Chaos and Open Quantum Systems

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A link between properties of quantized chaotic systems and random matrices will be reviewed. Dynamics of a closed quantum system can be represented by a unitary evolution of a pure state in the Hilbert space. We consider the case of a finite dimensional Hilbert space $\mathcal{H}_N$. Statistical properties of periodically driven quantum chaotic systems can be described by one of three circular ensembles of random unitary matrices, which belong to $U(N)$. The symmetry properties of the quantum system determine which of three universality classes – orthogonal, unitary or symplectic should be used.

To describe the effect of a possible interaction of the system in question with an environment one needs to work with density operators, which are Hermitian, positive and normalized. Discrete time evolution of a density matrix can be represented by so-called quantum operation (completely positive, trace preserving map). We are going to review the canonical Kraus form of such an operation and its representation by the dynamical (Choi) matrix.

To describe dynamics of an chaotic quantum system, interacting with an environment we introduce an ensemble of *random operations* and discuss practical algorithms to generate them. We investigate spectral properties of the associated superoperator $\Phi$, which sends the set of quantum states of size $N$ into itself, and state a quantum analogue of the Frobenius-Perron theorem concerning the spectrum of stochastic matrices.

We derive a general formula for the density of eigenvalues of $\Phi$ and show that for large $N$ they are described by the real Ginibre ensemble of random matrices. We analyze the size of the spectral gap, which implies that a generic state of the system converges exponentially to an invariant state.

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